

### 18.5.3. (2) Sharing ("Duality - Crystals")

#### 18.5.3.1. at. The last part - or the deaf ears

**Rating** 170 (i) (28 February) I come to the third of the "four operations" around my mathematical work tick (waiting for the fourth in the next note, retracing the work of Zoghman Mebkhout).

III The operation "Duality - Crystals" (or: "The Beautiful Remains ...").

As I see it now, it is roughly speaking a **sharing** part of my work concerned

ing cohomology which was not yet appropriate (de facto or symbolically) by **P. Deligne** <sup>541</sup> (\*).

This one obviously has reserved the lion's share, with the motives and cohomology étale, and more specific- the cohomological tool l-adic. The sharing of the rest (\*) is done between two other of my students homo

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gists, **JL. Verdier** and **P. Berthelot** <sup>542</sup> (\*). The consensus is established, I can not say when and p. 921

how, seems to be the following: Berthelot has all the crystalline cohomology, and the rest at Verdier, which ix basically everything that revolves around the duality of yoga <sup>543</sup> (\*\*), and yoga derived classes and triangulated which constitutes the algebraic prerequisite.

Regarding Berthelot's participation in the sharing of my remains, I have only one fact, of

size it is true. I stumbled upon it last year, during the reflection in the note "Les co-

heirs. . . "(N ° 91) and I spent a small sub-notes (n ° 91 1). This is the article-survey Berthelot

that I quoted <sup>544</sup> (\*\*\*), with the main ideas for a "synthesis" (he said) the cohomology Dwork-

Monsky-Washnitzer and crystalline cohomology, at the September 1982 Luminy Symposium entitled

"P-adic analysis and its applications". In the introduction, part (b), it gives a short history of co-operation crystalline homology, in a narrow mind that does not correspond to the vision much more

I had extensive yoga lens <sup>545</sup> (\*\*\*\*).

My name is missing from both the text of the article and the bibliography. I refer to the sub-note cited for some comments and precisions, which need not be repeated here. I would only add that once my nobody removed from the painting, it's none other than him alone, Berthelot, who is the father of cohomology crystalline, without even having to bother to say it in the clear - a certain style of appropriation visibly school ... This is his thesis indeed, that he prepared with me from my starting ideas, which is the first published work on the crystalline theme (apart from the very sketchy outline that I myself had

<sup>541</sup> (\*) (1 May) must, however, set aside the duality formalism in **coherent** context, which (unlike a impression that has proved hasty) has apparently not been appropriate yet by any of my student cohomologists, nor by no one else to my knowledge. It is true that the only reference text, exposing most of my ideas and results on this topic, is "Residues and Duality" by R. Hartshorne, which allows to refer to it without having at any time to pronounce a unwanted name. . .

<sup>542</sup> (\*) (1 May) It appeared since he should add a "fourth thief" in the person of Neantro Saavedra Rivano, who appropriates the philosophy of the motivational Galois group, via the categories named "for the circumstance" Tannakian ". But he is simply a "father of straw" on behalf of Deligne, who "recovers" paternity ten years later.

For the detailed story, see the following notes "The Sixth nail in the coffin", n ° s 176 1-176 7.

<sup>543</sup> (\*\*) See page footnote from previous page.

<sup>544</sup> (\*\*\*) rigid geometry and cohomology of algebraic varieties of characteristic p, Pierre Berthelot Symposium in Luminy September 6-10 (CIRM) "p-adic analysis and its applications".

<sup>545</sup> (\*\*\*\*) see in this respect the sub-note "Deaf ears" (n ° 170 (i) bis) which follows this note.

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### 18.5. THE FOUR OPERATIONS (on a body)

made of some of the ideas of

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starting <sup>546</sup> (\*). His thesis presents a work of large foundations to p. 922

first part (170 (i) bis) at least in the program I had proposed to him.

This memorable "survey" takes place in 1982, one year after the "Pervers Symposium" (Luminy June 1982), which he will be discussed with "Operation IV". I did not bother to go back to the prints besides Berthelot

in my possession, to learn if this participation in my Burial represents a late turning point

in his relation to me and my work, or if it is the continuation of an older attitude. In

the first case, it's a safe bet that this turn comes in response, in a way, to self-escalation

sudden and unbridled in the general degradation of scientific ethics, accomplished the year before with

the Symposium. I recall in this connection that the same year 1982 is also indicated by the publication of the

"memorable volume" LN 900 exhuming the reasons <sup>547</sup> (\*\*), where the one who made the transaction costs is not a vague "unknown service" (as during the brilliant Colloquium), but a "deceased" whose name, despite everything, remains in the memories (albeit with regret ...). The operation of the previous year had shown well enough

clear that no more restraint was required - and the "Reasons operation" was indeed "Operation Crystals" and all those that preceded, without making the slightest wrinkle. . .  
**Rating** 170 (i) a (170 (i) bis) (28 February and 30 April) <sup>548 (\*\*\*)</sup> I mean here by "first phase" of the theory crystalline (in  $p > 0$ ) crystalline cohomology, with constant coefficients (or "constant twisted"), **clean and smooth** patterns on a database schema for  $p$ . You just have to work with "ordinary" or "infinitesimal" crystalline site, which I had introduced (provisionally) towards the end of the sixty <sup>549 (\*\*\*\*)</sup>. In fact, contrary to Berthelot's restricted meaning in giving the term "cohomology-crystalline logic", it had for me from the beginning a much

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wider, I have hidden  $p$ . <sup>923</sup>

him or anyone, and that my students have apparently forgotten - to "reinvent" a small piece of ten or fifteen years later. . .

On the one hand, my crystalline ideas, from the beginning, were by no means confined to the case of schemas of a particular character.

given  $p > 0$ . My first crystalline reflections, before coming to me the new idea of introduce "thickened with divided powers," were placed on **zero characteristic** patterns where the divided powers are present automatically (and for that, tend to go unnoticed ...).

The natural culmination of this direction of research, renewed thanks to the ideas of Zoghman Mebkhout, will be the formalism of the six operations for the "crystalline coefficients of De Rham-Mebkhout" on the character of nullity (to begin with), formalism to which I already alluded in the note "The melody to the tomb. - or sufficiency" ( $n \circ 167$ ) In the sixties, I glimpsed a crystalline cohomology gallery without distinctions of characteristic, in the form of a crystalline formalism of the "six operations" in the context (for example) finite type schemes on the absolute basis  $Z$ . It had to encompass crystalline theory "ordinary" (which still looking - and still seeks) for type schemes ended up on the field  $F_p$

to  $p$  elements. I am convinced that it is to have forgotten and buried this vision of the deceased master (yet simple and inspiring as possible), which is the cause of the desolate stagnation of crystalline theory, nearly twenty years again after the vigorous growth of its beginnings.

<sup>546 (\*)</sup> The only sketch published these ideas, after five presentations I had given at IHES in November and December 1966 written by I. Coates and O. Jussila, is "Crystals and the Rham Cohomology of Schemes", in Ten lectures on Cohomology Schemes (North Holland, Amsterdam 1968) pp. 306-358. All essential ideas for starting are sketched out, including including the need for local thickening of the Monsky-washnitzer (pp. 355-356).

<sup>547 (\*\*)</sup> See "Silence" (No. 168  $\circ$ ), including "... And exhumation" ( $n \circ 168$  (iii)).

<sup>548 (\*\*\*)</sup> This sub-rating comes from a footnote page to the previous note "The share of the latter." (\*\*\*\*) (May 12) In fact, it is already in 1966, see the note of b. from p. (\*) above.

<sup>549 (\*\*\*\*)</sup> (May 12) In fact, it's already in 1966, see note b. from p. (\*) above.

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On the other hand and to return to the approach of Monsky-Washnitzer, who had helped to "trigger" on crystalline cohomology, from the very beginning I had in mind the need to introduce (for care of a theory that would not apply only to clean and smooth patterns) a larger crystalline site the "infinitesimal" site, where the "thickening" would be considered spectra of **topological** algebras (with ideal to divided powers) suitable, perhaps those used by Monsky-Washnitzer (cleared useless hypotheses such as smoothness) (\*). Identify "good site" <sup>550</sup> 0

and "the right coefficients" is part of the  $p$ . <sup>924</sup>

gram that I had bequeathed (to no avail, it now appears) to my cohomologist students, to begin by Berthelot. Having thought about it lately "by the way" (on the occasion of the writing of Harvest and Seeds), and reminding me of the imperative of a crystalline theory encompassing all the characteristics at the time, I have come to wonder if these topological algebras (at the Monsky-Washnitzer, or any other reasonable variant) are not, too, too "coarse" (in the same way as the formal series because they are too far from the algebraic, and if there is no need to replace them by which are (in a proper sense) "neighborhoods étales." I think to return to these questions in the part of Reflections following Harvests and Seeds (volume 3, I presume), with the presentation of the yoga of six operations and the problematic of the coefficients, and in particular the crystalline coefficients of the type Rham-Mebkhout".

Mebkhout had also sensed that his philosophy of 3-Modules should provide a new point of view. calf for crystalline theory. But his suggestions in this direction, Berthelot especially in 1978, coming from a vague and unknown grothendieckien unrepentant, fell on deaf ears <sup>551 (\*)</sup>. . .

<sup>550 (\*)</sup> As I specify it in a previous note b. from p. (see page 922), there is talk of such thickening at the Mons-Washnitzer in my first and only lecture on crystalline yoga, late 1966. From that moment, it was clear to me that the crystalline cohomology of characteristic  $p > 0$  was going to be played for the most part on rigid-analytic spaces of null characteristic. Of course, I did not fail to let everyone know that it could concern, and first and foremost

surely place to my student Berthelot, once he had chosen to invest in the crystalline theme. In the article quotes, next a style that I recognize well and Berthelot did not invent, it looks like he just discovered (fifteen years later) the unsuspected connection with the rigid-analytic geometry (x). He poses to the brilliant inventor a "common generalization" (from the theory of Monsky-Washnitzer and crystalline), which he pompously calls "rigid cohomology" (and which will be called soon, as it should be, "cohomology of Berthelot"). I also note that Berthelot's work is "the continuation of a reflection conducted with Ogus" - the same Ogus who distinguished himself the same year (1982) by his participation in the scam "Motifs", as co-author of volume LN 900.

The systematic burial continues in a later article by Berthelot (of which I have a preprint) "Cohomology rigid and Dwork theory: the case of exponential sums" (undated) No reference to the deceased for the crucial notion of F-cristal, or that of cohomology with its own support (which I have the honor to introduce into algebraic geometry in February 1963, twenty years before. . . ). These notions are so natural, moreover, that there is really nothing to worry about. . . The concept generic fiber of a formal scheme (above a discrete valuation ring), as a rigid-analytic space, is generously attributed to my ex-student Raynaud. This notion was known to me before neither Berthelot, Raynaud nor elsewhere no one else has yet heard the word "rigid-analytic space", since it is the need to be able to define such a generic fiber that was one of my two motives for predicting the existence of a "rigid-analytic geometry", and that he too was one of Tate's two lead wires, putting together a construction in the shape of a geometry: its definition had to be such that the notion of "generic fiber" becomes tautological. . .

(x) (September 1985) In fact, the first to predict the existence of such a theory was J. Tate, in August 1959. See on this subject the Note No. 173 ° d) ("The Burial - or the natural slope"), especially the footnote at page 1132 page. 551 (\*) To have deaf ears does not prevent the same Berthelot, in the article I cited in the previous footnote b. from p. refer casually (at the end of s. 3 A) to "an analog of  $x$  of the 3-modules theory on a complex manifold" whose "for the moment" one does not have yet in the rigid-analytical frame. No question of course here to mention the name of a certain unknown wave who had come to make wacky suggestions four or five years before, and this especially some Symposium last year (which will be discussed in the following note "Apotheosis", n ° 171) had given clear the tone regarding the unknown wave in question, surely, in a few years time, and with the blessing of the real father of the well-known philosophy of "Riemann-Hilbert-Deligne", Berthelot will be the brilliant inventor of philosophy 3-Modules in the context of "rigid-analytic cohomology", also called (even if it itself does not name it thus) "cohomology of Berthelot". Like what, these days, there is no need to have a very fine ear for yet go far. . .

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18.5.3.2. b. **Glory galore - or ambiguity**

Note 170 (ii)

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(28 February) 552 (\*) To locate the "Duality operation," the dubious benefit of JL Verdier, there p. 925 would be a good start to say a few words about the yoga of duality (called "six operations" - but the name sank without a trace) that I had developed from the second half of the fifties, and that of derived categories, which is truly inseparable. I expressed myself in a rather detailed way topic in the note "My orphans" (No. 46 °, especially pages 177-178) and in the sub-46 ° footnote 2 to this (pages 186-187), and finally (in a beginning of reflection on the role of Verdier in the burial of my point homological algebra of view) in the note "The instinct and fashion - or the law of the strongest" (n ° 48). He is seems pointless to return to, and I suggest the reader refer to it as needed, before continuing with the narrative operation "duality" 553 (\*\*).

Verdier's attitude in the operation of sharing appears more ambiguous than that of his two friends, **i played the**, sometimes simultaneously, **on two tables** that may seem contradictory. I had a hard time myself, first, to recognize myself, so confused was the situation. On the one hand, after his defense in 1967 and especially after my departure in 1970, he strove (for reasons that escape me) **bury and discredit** the cohomological algebra yoga and duality he held me, while had devoted most of his energy, throughout the sixties and until the defense his thesis, to develop these ideas and enrich them with his own contributions. On the other hand, from all less than 1976 (nine years after the defense of his thesis-sic), and with the encouragement and effective support of Deligne, he pretends to **assume** credit both initial ideas (to the extent that they did not remain boycotted), of all the methods and results that I had developed

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around the p. 926

theme of duality spreads, methods that apply mutatis mutandis in all sorts of contexts 554 (\*)  
such as topological spaces, or complex analytic spaces.

Regarding Verdier's attitude towards derived categories alone, I tried to put my finger on the sense of this ambiguity in the note "credit Thesis and comprehensive insurance" (n ° 81) 555 (\*\*). There will be also a number of material facts, particularly concerning the strange circumstances surrounding his thesis work (still not published today) and the defense. Looking back one year, the vision things that emerge during this reflection probably seem correct to me (perhaps), but nevertheless superficial. It is clear to me that the **true** motivations of Verdier do not place themselves at the level of any derisory "calculation of returns", but that they are of any other nature, and essentially involve his ambivalent relationship to me. Even for an observer superficial, it seems to me, it is particularly flagrant in his case that believing to bury the one who was his master is none other than **himself** and the creative force within him he buried every day and up still.

552 (\*) The text of the note was taken to the net, and ground on some points, 1 May (day lily).

553 (\*\*) (May 12) See also Note "ancestor" (n ° 171 (i)) and "The tour of building sites - or tools and vision" (No. 178 °), including construction sites and "six operations" "coefficients" (n ° s 3.4).

554 (\*) Of course, in the "other settings" in question, the context of starting trouble spreads, namely the need for a "breakthrough" which gives a minimum of hold over the étale cohomology (in the absence of transcendent constructions well known singular simplexes, retraction methods, etc.) do not arise. My students have all found situations where heavy work "breakthrough" was already accomplished by another - they only had to bring their furniture, in short, that often "the other" provided them over the market. As soon as the occasion came, they hurried to bury him, to take advantage of what they have seen fit to appropriate, and make fun of the rest. . .

555 (\*\*) In writing this note, I was not aware yet of how Verdier had distinguished himself with "good reference "that it provides in 1976 - see" step 2 "below.

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To go around the operation "Duality", I will now make a short retrospective of the different steps that are familiar to me from this operation, and more generally, from Verdier's participation in the surely.

**Stage 1** (1966-1976). It's after my departure in 1970, I can not say when exactly, that Verdier informs me that he no longer intends to publish his thesis. I remind you that this one was supposed to present the new foundations of homological algebra, in the context of derived categories. In my eyes, the reason for being of his thesis work was to be made available to all, to provide a reference text comparable in scope to the Cartan-Eilenberg book, directly adapted to the new needs

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during the fifties and sixties in the wake of my works and those of my students. In hindsight,  
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I realize that this new cohomological language was not yet fully assimilated (and again, I would say even today. . . ) that by my cohomologist students, and that Verdier's decision was equivalent henceforth to draw a large line on this new vision of homological algebra. So also, his "thesis" of twenty-five pages, which was confined to presenting a convincing sketch of ideas of which he himself that they were not due to him, lost his meaning and became, strictly speaking, a "phony thesis". But to beginnings of the 1970s, by learning (with surprise) Verdier's decision, I was absorbed so intensely in tasks at odds with my mathematical interests of yesteryear, that these questions were then for me infinitely far away. The idea did not come to me to ask about the thing, learned in draft (I can imagine me) between a public debate on the scandal of fissured casks of atomic waste at Saclay, and a working session for writing *Surviving and Living!* And even less, would I have thought of react. The first time I "pose" finally on the meaning of this act of Verdier, and where his nature of deliberate sabotage shyly begins to appear, is in the note already quoted "Instinct and fashion - or the law of the strongest" (n ° 48) resumed a few weeks later, after the discovery of Burial "at its best"

in the much more detailed and thorough notes "Thesis credit and insurance all risks" (n ° 81).

In retrospect, it becomes clear that Verdier's division in the work he had himself assigned, and was part of the "contract of good faith" he had spent with his thesis committee (see note cited n ° 81), must go back at least to 1968 or 1969; otherwise the writing and publication of his "thesis" would have been done well before my departure in 1970. I remind you that I had submitted the work program on his thesis as early as 1950, and that for a gifted and motivated researcher as he was then, this program, with an extensive drafting of new foundations, should hardly represent three or four years of work to break everything, get in touch and everything. It is also true that a certain mentality, which consists of withdraw in advance a credit for a planned "work", so that there is no longer any reason to tire - such a mentality now becomes apparent to me after 1964 already, with the vicissitudes of the formula known as "de Lefschetz-Verdier", and later, with duality (known as, of course, "de Verdier")

locally compact spaces, in the spirit of the six operations

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(which remain unnamed) <sup>556</sup> (\*). But

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throughout the sixties, locked up that I was in my tasks and in the vision that I tirelessly pursued through them, such as the elusive and ubiquitous white whale of Ahab, I was miles away from to doubt me that something "was wrong" in him who was for me like a close companion in tasks that I thought were "common" - nor would I have suspected it for any of my other students cohomologists. And with twenty years of hindsight, I am seized now to see how, for ten years of my life (if not fifteen or twenty) I lived completely **out of step** with the reality around me, and this, not only in my family life (where I have come to notice for a long time), but also

<sup>556</sup> (\*) See, on this very special spirit, the sub-score "Heritage - or shenanigans and creation" (n ° 169 6a) and also sub-notes of the last year (s n ° 81 2, 81 3) to the already cited note "credit Thesis and comprehensive insurance."

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in professional life, in which I invested with passion. . .

But I come back to "step 1". Verdier's ambiguous relation to my person and my work appears in all case after the completion of the SGA 5 seminar in 1966: no more than any of my cohomologists, he feels involved by writing this seminar <sup>557</sup> (\*\*), which remains in the hands of "volunteers" - sic overwhelmed by the task, or unwilling to keep their commitments. Obviously, from that moment already, the situation in all of my student cohomologists is rotten, though I do not see of nothing, preferring to live in a world where everything is order and beauty. . . It's eighteen years later that I begins to throw a first and timid look at what really happened, in those times that (there is a once again) had seemed to me idyllic <sup>558</sup> (\*\*\*)).

After I left in 1970, and already before he told me his "official" decision to scuttle his work foundations, Verdier's ambiguity in the sixties is confirmed by a connivance with various mini-scams of the growth of his friend Deligne, which he could not fail to realize:

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p Of retraction. 929

my person in articles Hodge I, II, III <sup>559</sup> (\*), and in the published version of the monodromy seminar SGA 7 II (presented under the names of Deligne and Katz, the latter unexpectedly taking the place still hot of a deceased. . .). The same year (1973), he was also able to miss the Mac article Pherson, where is solved a "conjecture of Deligne-Grothendieck" which he knows that Deligne is for nothing.

Until 1976, Verdier's role in the funeral seems mostly passive, as far as the operat-tacit annexation at least. On the other hand, by refraining from publishing what was supposed to constitute his thesis (which had been granted "credit" <sup>560</sup> (\*\*)), he played right before I left a crucial role in the in-from my point of view in commutative homological algebra (which he had adopted for a time), and its use as an "everyday" technique both in algebraic geometry and in topology and in algebra. Like his friends Illusie and Deligne, thus scuttling the work with his own hands, for the the pleasure of burying the one who inspired him, he deserved the unconditional recognition of the Congregation unanimous. . .

This deliberate burial was also clearly expressed in its discouraging attitude towards Zoghman Mebkhout, after 1975, when he pretended to be inspired by my yoga of duality, and that of the derived categories. On this subject again, I refer the reader to the more detailed notes already quoted "My orphans", "Instinct and fashion - or the law of the strongest", "credit Thesis and comprehensive insurance" (n ° 46, 48, 81) and Note "The unknown service and the theorem of God" (n ° 48 §<sup>561</sup> (\*\*\*)).

**Step 2** (1976). In 1976 the publication of the "memorable article"

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Verdier in <sup>562</sup> Asterisk (\*), p. 930

already mentioned as "episode 3 of an escalation" with the operation "Cohomologie étale" (see note "maneuvers" n ° 169). I remind you that this fifty-page article consists of (apart from a few pages of his own) to take back text to a number of notions and techniques that I had developed <sup>557</sup> (\*\*). In retrospect, I have to wonder what Verdier could well use his time from 1964 (where he finished in my contact, by getting into the bath of new cohomological techniques) and 1970, while he does not deign to grab and to carry out any writing task, not even theories of which he would present himself as the author. For the list of his contributions, valid, but none of which are completed, see sub-81 ° footnote 1 to the much quoted note.

<sup>558</sup> (\*\*\*) see in particular, in "Fatuité and Renewal", the "A world without conflict?" (n ° 20), where only the point of interro-in the name of the section may suggest some doubt about "idyll".

<sup>559</sup> (\*) In the joke "complex weight" (see the note of the same name, n ° 83), I really thought discern an allusion on the tone

of the challenge, the most obvious swindle I know about one of my student cohomologists, that of Deligne in his 1968 article on the degeneration of spectral sequences. If I only saw fire then, the example given by my most brilliant student has not yet been lost for everyone!

560 (\*\*) See note already cited n ° 81.

561 (\*\*\*) (1 May) Also sub-note "Hatching a vision - or the intruder" (No. 171 ° 1) in note "Apotheosis".

562 (\*) JL Verdier, "homology class associated with a cycle" n ° 36 Asterisk (SMF) p. 101-151 (1976).

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ten years before in SGA 5, and this without hinting at my person or at a seminar dealing with these things. This publication, which I discovered a year ago in the wake of the Symposium Pervers (in the note "The good reference "n ° 82), then lit a completely new light towards the reluctance of himself and my other cohomologist students, to put the SGA seminar 5 (under this name, and with paternity that is to the mathematical public.

There is no need to come back here to comments about this article, which I made in yesterday's note already cited (n ° 169). As a funny detail, I will only add that this is the manuscript of this "work" (sic) of Verdier, that he had the kindness to communicate to Zoghman Mebkhout the previous year (1975), which been to this Sesame-opener You cohomology of varieties, and the basis for admiration without reserve one which now was figure "benefactor." This admiration was also hard skin, and has finally disintegrate completely, I believe that following the misadventures of Zoghman occasionally Pervert's Symposium.

Deligne said 563 (\*\*) he has taken note of Article Verdier after the publication of "SGA4 1

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(Sic) and SGA 5, the following year (1977) - which would run counter to my belief that the publication of "good reference "Verdier scored a last essential step in the" escalation "of scams, who finished eventually lead to the operation of any other major "SGA4 1

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- 5 USG "the following year On reflection, I

hard to believe the version of Deligne. He is one of the best informed mathematicians I know, and remained in close contact with Verdier always, it is hardly possible that he was aware Project already Verdier, than it has received a preprint (and this even from before Mebkhout), and it has was one of the first served for reprints in 1976. This article filled (as confirmed to me Deligne himself) a gaping hole in the literature (for lack of publication of the seminar SGA 5 after 1966) and it is hardly possible nor Deligne have taken the

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worth at least browse the - issue a quarter

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Time to break everything to someone "in the know" as her 564 (\*). Anyway, the fact that plagiarism manifesto has been no reaction from any of the six or seven other former listeners that SGA 5 were well "in the know", was well assured of collusion burr between all concerned. The time was ripe for the slaughter rule in the mother-seminar SGA 5, and to shatter my work on cohomology spreads. . .

**Step 3** (1977). In this operation "SGA4 1

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- SGA 5 "which takes place in 1977 on the initiative of Deligne and with the participation eager to Illusie Verdier played this time a supporting role, contributing to lean Issue the misleading name "SGA4 1

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"A" State 0 "of his thesis-sic (disappeared, her body well ...)

exhumed specially for the occasion after a sleep of fourteen! Anywhere in the volume, that Whether in the introduction that this text rabiot ("become untraceable" - and for good reason) is properly mounted pin or in the text itself, there is a hint at a role I have played in the ideas that are there developed; nor, moreover, the fact that this text was a day destined to become a thesis. No more that Deligne, Verdier has seen fit to inform me of this publication (and for good reason, yet), nor to me send a copy of trompe l'œil volume. I refer, for details, Note "The comrade" (n ° 63 " © written in the excitement of exploring this exhumation on the sly), and further reflection in the already noted repeatedly cited "credit Thesis and comprehensive insurance" (n ° 81).

Thus, ten years after his defense of unusual thesis, Verdier took the opportunity offered him Deligne

563 (\*\*) See footnote "The dot the i" (n ° 164), Part IV 1.

564 (\*) I can imagine also that much stronger than the mathematical interest (as this article has nothing to learn Deligne, he does not already know as auditor SGA 5), had to be the one to be able to learn first hand and black on white, burr retraction of the late master, according to tradition he had himself inaugurated for already

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to take, in short, an " **option** " on an uncontested paternity and undivided perspective "categories derived "in homological algebra, with the full guarantee of its prestigious friend, and this at a time where either still continued to maintain a **boycott** of fact on the use of this same point view <sup>565</sup> (\*\*). The boycott, which weighed heavily on the work of Zoghman Mebkhout, sentencing him to a complete solitude, remained in force until "Pervert Symposium" in 1981.

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Thus, in 1977 Verdier appears as the father-in-reserve a yoga cohomology which, for the moment, <sup>p. 932</sup> remained subject to a tacit disdain fashionable - but you never know. . . Moreover, since the previous year, with the publication of "good reference," he was father figure of part of the duality formalism developed by me (on homology classes and cohomology "discrete" cycles associated with the formation bidualité of ism, theorems of finiteness Version constructability etc.) - not to mention the dual spaces locally compact, which remained, too, in an ambiguous status, waiting status - like yoga Derived classes that gives it meaning.

**Step 4 ( Symposium Pervert , June 1981).** This is, by far, the culmination of the participation of Verdier the funeral. This symposium devoted shameless spoliation Zoghman Mebkhout pioneer point unifying view and fruitful 3-ons in the cohomology of algebraic varieties. As qu@rgani-Official sateur Symposium, with B. Teissier, Verdier plays a leading role. More on that in the next note with the "operation IV" (called "Symposium Pervers" or "unknown service"). Here I borrow to direct benefits for Verdier, under the "sharing" of an inheritance (where the deceased bequeaths rest studiously ignored. . .).

This symposium devoted to "return" triumphant derived and triangulated categories in the Arena mathematic. As the "father" of those categories (he had done for fifteen years to bury) is Verdier, after Deligne, which appears as the main hero of the happening. That is the impression that at least emerges from the main section of the Symposium, from the pen of Deligne item constitutes the volume alone I and the centerpiece of the Proceedings of the Symposium <sup>566</sup> (\*). Coincidentally, the skeletal and providential "State 0" of a thesis (which I never would have dreamed accepted as a doctoral thesis, which had come bail timely pirate text "SGA4 1

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a little thin in the armholes) - these become the brilliant piece <sup>p. 933</sup> conviction, allowing the father-to-the-sly Verdier, in a cloud of references to "SGA4 1

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"To swagger

modestly as providing precursor big rush-called "perverse sheaves" (which are there for nothing yet) and a new and late re-start of the cohomology of algebraic varieties (on broken an unknown wave that nobody dares to pronounce the name. . .).

The same article (signed Beilinson-Bernstein-Deligne) spends returned to power, also, the formation six operations ism (never named, of course) in the context spreads, with the notation now devoted created that I introduced in the fifties. As I wrote elsewhere <sup>567</sup> (\*) "there is not a page Article city. . . which is deeply rooted in my work and bears the mark, and this far in the logs that I introduced, and the names for the concepts involved in each not - that are the names I had given them when I met them before they are Named ".

<sup>565</sup> (\*\*) As I explained in a previous note b. p. (Note to the page), in the coffin text-named "SGA 4 1

2 "Deligne

could not avoid the use of derived categories in demonstrating "the" formula. This is probably what has been suggested the idea of expanding its volume with the "state 0" of a shipwrecked thesis. In fact, it did not change until 1981 the situation boycott on derived categories.

<sup>566</sup> (\*) Acts published in Asterisk n ° 100 (1982) - under the title "Analysis and topology of singular spaces." In fact, the Acts in question dated from 1982, have been completed to print in December 1983 and Mebkhout became aware in January 1984.

<sup>567</sup> (\*) See note "The Iniquity" (n ° 75), p. 288.

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was still learning the ABCs of cohomological language is renamed "Verdier duality" in eupho-  
General Series 568 (\*\*). Its prestigious protector not going to skimp on the little, these days of jubilation! The name  
the deceased does not appear in Article 569 (\*\*\*), nor in the introduction to the volume, signed Teissier-  
Verdier. Nor the unknown wave (Zoghman Mebkhout, not to call it), without which the article nor any  
Symposium brilliant, would have seen the day. . .

For the slaughter, it was the slaughter! Aside from the reasons that would soon follow (from the year  
after), and possibly the lens yoga, sharing stories without the cohomological inheritance of a deceased  
ever named was now consumed thing, and this in unanimous agreement and **to the general satisfaction** .

### 18.5.3.3. c.the jewels

Note 170 (iii)

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(1 March) three "operations" which I reviewed in previous notes concern

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the "sharing" of the "legacy" that I left, as my written and unwritten work on cohomology  
schematics. "Beneficiaries" of this direct sharing were three of my five cohomologistes students  
namely Pierre Deligne, Jean-Louis Verdier, Pierre Berthelot and 570 (\*). But each of these three operations  
(Like the one that follows) could be done with the connivance (and sometimes active support) of a large  
many colleagues more or less "hip" on the cohomology of schemes, which include in  
first up my five cohomologistes students, including, besides those I have named, Luke  
Illusie and Jean-Pierre Jouanolou (\*).

These three operations, and he will be fourth question, strike me as inextricably linked,  
both in their underlying motivations, in their most tangible adventures. The first signs  
discrete back to the years 1966 to 1968, but its most glaring manifestations are placed after my  
"departure" in 1970. The departure and a general state of morals in the "big world" mathematical 571 (\*\*)  
created external conditions conducive for such a large-scale operation, probably unique in  
its kind in the history of our science.

This operation aimed firstly to **discredit** most major **key ideas** I had introduced  
math 572 (\*\*\*), and burying the **vision** unifying in which they were inserted; then, to discredit  
or retract the **role of the worker** in the creation of those among the tools I had shaped in the  
Dictation these ideas and inspired by the overall vision, which served as basic tools in the work of  
Deligne and my other cohomologistes students; and finally, in a last stage, to appropriate authorship  
568 (\*\*). In the index notation, the functor Dualising (which I introduced in the context spreads in 1963, and which also  
the subject of the presentation I-publishing SGA Illusie 5, where he managed to survive) is called "Verdier duality." This name  
reappears  
everywhere in the text (eg, pages 62, 103 -.. looking at happiness-the-lucky...). I swear I am not making!  
569 (\*\*\*) My name still appears in the bibliography, with the acronym EGA (it will come to substitute a text ad  
hoc someday. . . ). The name of Mebkhout is absent as well as the text of the bibliography. There©trace in all  
volume.

570 (\*) (2 May) should be made to add a fourth "beneficiary", I discovered recently only know Neantro  
Saavedra, which was discussed in a previous note b. p. (Note (\*) page 921).

571 (\*\*) (May 2) There surely had a share double meaning: a state of degradation of mentalities (in which I myself  
even participated before my departure) promoted escalating looting and débinage of my work by a group of my former  
Students whose growing cynicism surely contributed in turn to create more or less generalized state of corruption that I  
notes today.

572 (\*\*\*) (May 2) for details on this, see Note "My orphans" (n ° 45) and especially "The tour of building sites - or tools  
vision "(n ° 178).

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ideas and tools that have had the good fortune to be adopted by my students, or end up imposing despite  
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boycott p. 935

that they had posed to them 573 (\*).

This ended in 1982 with the publication of volume Lecture Notes 900, consecrating relearn  
pearance patterns on mathematical public square, under a narrow form (compared to the vision  
had emerged for me during the sixties) and under the fatherhood (implicit and obvious) Deligne.  
She finally found its epilogue the following year in "the Praise of Death" three-pronged served in the plate  
Jubilee of IHES, published to mark the twenty-fifth anniversary of its existence.

The discovery of the "mine" posed by these texts is May 12 last year, 574 (\*\*), in the note "The  
Funeral Eulogy (1) - or compliments "(n ° . 104) It continues almost five months later in note (n °  
105) which follows, "The Praise of Death (2) - or force, and the halo 575 (\*\*\*) I will confine myself here to remind them.  
few words the spirit and all the salt of the "Praise" unusual.



This brochure (among others) a "portrait gallery", formed short topos on different present and past teachers of the institution celebrating jubilee. In the text (of the pen Deligne) that is me devoted text that is supposed to evoke a work, the word "cohomology" or "reason" is not pronounced. The note "scheme" either, or any other theory that suggests that I would have developed or theorized I would have demonstrated and that might have been used. For cons, I am decked generously superlatives cage and other kindnesses sounding: "... gigantic task", "twenty volumes...", "greater natural generality. . . " 576 (\*\*\*\*)" large

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Terminology attention. . . ", "problems. . . in the line that La- p. 936

Cait. . . become too difficult. . . "It is the burial at great fanfare and under the spotlight by the "Compliment" very sent huge and bloated as the deceased in question to "honor" the memory, and along with a fineness in the comic insinuation that decidedly lacking in clumsy ancestor. . .

In the guidebook dedicated to Deligne (and reviewed by it), nothing that would make suspect that I am to something in "the" demonstration of the Weil conjectures ( "proverbial difficulty"), duly MON-tered hairpin. On the contrary, it is emphasized that "this result seemed all the more surprising" that had to be shown, so to speak, against a "series of conjectures" of my own (this decidedly Grothendieck did in fact never more!), which also (it is added, leaving no doubt as to what necessary to think) "are now as then unaffordable" (read: when I had the unfortunate idea state them. . . ).

These two portraits minute, and a third component that complements remarkably (in one sentence lapidary three lines 577 (\*)) are true gems, without doubt as unique in their kind among the Praise Corpse served with skill in honor of a "deceased" (still not died in this case!). They are searched, using the care they deserve, in three consecutive notes already cited (n ° s 104-106), and, 573 (\*) (2 May) I note among these ideas and tools that I had introduced, which were buried and have come to dominate despite the boycott introduced by Deligne and my other students cohomologists: derived categories, the patterns (narrow version it is true) and yoga classes Galois Poincaré-Grothendieck (renamed "tannakiennes" for purposes of the Burial) the formalism of non-commutative cohomology around fields concepts, wreaths and links (developed by Giraud from starting ideas introduced by me in 1955).

574 (\*\*) This is the day that had already revealed to me the massacre shamelessly from original seminar SGA 5, at the hands of Illusie and with the active support or the eager connivance cohomologists all my students, under the tender eye of the "Congregation in full". . .

575 (\*\*\*) For an unexpected widening of the reflection on the Eulogy Funeral, see also the following note "Muscle and tripe (yin yang bury (1)) "(n ° 106), which opens at the same time the long reflection" The key to the yin and yang. "

576 (\*\*\*\*) This French-breakfast negro is a truly priceless find, to evoke a comical way (and not mine...) The bloated and free bombinage a huge talker. . .

577 (\*) I discovered this third installment in the reflection in the note already quoted "In Praise of Death (2) - or force, and the halo" - and it seems to me at once heavier meaning than the other two combined! It was he who inspired the name "The strength and

halo "given to this note.

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under the dynamic lighting most penetrating of the "reversal of yin and yang", in note (higher a few weeks) "The funeral of yin (yin yang bury (4))" (n ° 124).

#### 18.5.4. (3) The APOTHEOSE ( "coefficients Rham and 3-Modules ")

##### 18.5.4.1. a.L@ncêtre

Rating 171 (i) (1 March and May 2-8 578 (\*\*)) in each of these partial "four operations" I distinguish guées in my early burial is Deligne who obviously acts as a conductor (or rather, Grand Officiant to

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Funeral), with the more or less active participation of my other four students cohomologists, and with the connivance of a significantly larger group of mathematicians, all well aware of the situation (which obviously is not done to displease them...). This "conniving group" takes impressive and almost unbelievable proportions in the fourth partial operations

I am about to present to review.

IV Operation "The unknown service" (or "Symposium Pervers").

It is the operation of a **appropriation the work of Zoghman Mebkhout** - the only mathematician (to my knowledge) who has assumed the risk, after my departure from the mathematical scene to figure "connateur Grothendieck ".

This operation continued over a decade, from 1975 to today. At the risk of repeating myself, I will first recall the historical context.

In the second half of the fifties, I had developed in the context of a formation patterns

ism of "coherent duality". These thoughts, motivated by the desire to understand the meaning and exact scope of the duality theorem of Serre in analytic geometry and especially in algebraic geometry 579 (\*), had been pursued in solitude almost complete, not having the good fortune to interest anyone but I 580 (\*\*). It is these reflections

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led me to gradually disengage the notion of derivative category,  
p. 938

578 (\*\*) (May 13) This note and the following four originally formed a single note, "The Apotheosis" (n ° 171), March 1. It also included the previous note "Jewels" (n ° 170 (iii)). It was taken up and expanded considerably between the 2 and 8 May, especially as regards the mathematical part, and split into four separate notes "ancestor", "The work..." "... and the bargain", "The day of glory" (n ° s 171 (i) to (iv)), in addition to the note "Jewels" already mentioned. He added to it the eight sub-ratings (n ° 171 (v) to (xii)) relating to the four notes in question, and the four sub-ratings (n ° 171<sub>1</sub> to 171<sub>4</sub>) of April, making the story of the strange mishaps My friend Zoghman struggling with the "law of the underworld", as he me told himself. It is all of these sixteen notes (n ° s 171 (i) to (xii) and 171<sub>1</sub> to 171<sub>4</sub>) that now constitute the part "Apotheosis" in "Four operation" (which is the fourth said Apothéose and - until further notice - last. . . ).

579 (\*) My first thoughts were placed duality within the analytic spaces, and predate those of Serre. By using duality techniques "évêtesques" and Lemma Poincaré-Grothendieck on  $\hat{A}$  operation (I had just show), I proved that if  $X$  is a Stein manifold,  $H_i(X, \mathcal{O}_X)$  (resp.  $H_i(X, \omega_X)$ ) are Fréchet spaces nuclear, in perfect duality with  $H_{n-i}$

!  
 $(X, \omega_X)$  (resp.  $H_{n-i}$ )  
!

$(X, \mathcal{O}_X)$ ). I did not think then to apply the same method to the case of vector bundles (not being myself not see this fact very simple algebraic, that is opérateur  $\hat{A}$   $\mathcal{O}_X$ -linear, extends to differential forms differentiable with values in a vector bundle holomorph), nor to complex manifolds other than Stein (only then that are familiar to me). The demonstration of his Serre duality theorem analytically in the general case, is practically the same as that which I had found in a particular case.

580 (\*\*) Of course, the mathematician among all whom I would have expected an interest in my thoughts coherent duality, Serre was. He was interested, I seem to recall, by its widespread duality result in a coherent beam  $F$  (not necessarily locally free) on smooth projective  $X$  over a field  $k$ , identifying the dual of  $H_i(X, F) \text{ Ext}_{n-i}$

$\mathcal{O}_X$   
 $(X; F, \omega_X)$ .

This gave an intrinsic geometric sense to a result "computational" FCC (which was of course intrigued and inspired) where  $X$  is the projective space. But apart from this, one of the first in my journey of discovery of duality, and yet close to what was familiar, Hair has always refused to listen, when he took me want to talk to her duality. I think I hardly tried to talk to anyone else, other (much later) Hartshorne, who made my ideas on a

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whose objects were as "the coefficient" in the natural homological formalism and cohomological spaces and varieties of all types, fitting into a first embryo of a formalism of "six operations" on ringed spaces (pending topos ringed). Four of these operations were already more or less familiar since my work from 1955 "On some points of homological algebra" 581 (\*) the language of derived categories closely: with the notations that have emerged over the years following (along the view categories derived), it is the "internal" operations

The  
and  $\otimes$

Rohm (Version "functor total derivative" of gold beams formalism  $T_i$  and  $\text{Ext}_i$

introduced into "Tohoku"),

and "external"  $Lf^*$  and  $Rf^*$  (inverse and direct images "on Leray") forming two couples of functor (or bifunctors) assistants. If  $f$  is a morphism "immersion"  $i: X \rightarrow Y$ , there it still adds

couple assistants functors  $Ri_! Ri_!$ , Embodying respectively the operations of "zero-extension" and "local cohomology racks in  $X$ ". The common thread in my thoughts is to arrive at a **theorem**

**duality** (overall, at a time when there was no local version yet...), generalizing the proven

by tightening to a locally free coherent beam onto a smooth projective variety over a field. It was

to give a formulation which would apply to any coherent beam (or such complex) or

even a quasi-coherent beam without assumption of smoothness or projectivity  $X$  (keeping only

cleanliness, which seemed essential 582 (\*\*)). Moreover, in analogy with my thoughts on the theorem

Riemann-Roch, I felt good statement was concern, not a variety of a body, but a mortal own morphism  $f: X \rightarrow Y$  patterns, by any elsewhere. It approximations

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successive at p. 939

During several years of work 583 (\*), the global duality theorem gradually settles in his superfluous assumptions, along with the notion of derived category also out of limbo tipped to take concrete form, and give the formalism and set an **intrinsic sense**, failing which I would have felt quite unable to work! It is first to reach a clear statement global duality that fully satisfies me, I introduce the formalism of **complex dualisants** and clears the **bidualité theorem**, and I discovered (under suitable assumptions noethériennes) the existence of a complex Dualising injective essentially canonical, I call the "**residual complex**" and variance theory to it. A first formulation of the overall duality theorem, which at one time seemed to be "good" was that the functor  $f^*$  was switching to  $X$  and functors dualisants  $Y$  (for two complex dualisants that "correspond"). It was only later that I discovered the variance theory for complex alone dualisants (via residual complex) is spreading through a completely new kind of functor, the functor  $f_!$  where "unusual mirror image" of a local nature  $X$ . Therefore, also appears the final wording of the duality theorem for proper map  $f$ : this new functor is **right adjoint** to  $f^*$ , so fitting in a series of three adjoint functors  $Lf^*$ ,  $R^*$ ,  $R_!$ .

To have a fully completed formalism, it only lacked a description of a functor  $f_!$ , beautiful seminar at Harvard, published in 1966 ("Residues and duality" by R. Hartshorne, Lecture Notes in Mathematics, n° 20 Springer Verlag).

581 (\*) In Tohoku Mathematical Journal, 9 (1957), p. 119-221.

582 (\*\*) On this subject the note b. p. (\*) Page 940, below.

583 (\*) It goes without saying that during the "years of work", I had many other irons in the fire that the only issues consistent duality! I am so familiar with the fundamentals then known algebraic geometry (with point Serre to FCC as a main reference), with the issue of the Weil conjectures, and formalism intersection multiplicities learned in a course of Serre, where he developed his idea of the "sums of alternating tor"). this would trigger me in 1957 on the K-theory of formalism and the theorem of Riemann-Roch-Grothendieck, very close (by its mind) duality of my reflections.

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"Direct image to clean surfaces", for a morphism (separated) of any finite type, generalizing functor  $f$  is already known when dipping, reducing to  $Rf^*$  proprietary  $f$ , and forming with  $Rf_!$  a couple functors  $Rf$  assistants  $!Rf_!$ . I do not remember being afflicted in the fifty years of this imperfection of a formality whose general, beyond the schematic consistent duality or analytic,

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still eluded me 584 (\*).  
p. 940

This gap appears to me fully only in 1963, when I discovered that in the context of cohomology spreads (to "discrete" coefficients) just born, is a formality in all points similar the coherent formalism, with more precisely a functor  $f_!$  ((direct image to clean surfaces) defined for **any** separate morphism of finite type. Moreover, it is not guiding me step on the work I had done in the coherent case years ago (without interest anyone other than me), then I come (in Within a week or two to break everything) from the two base change theorems-key, to establish complete formalism said "six operations". This is a duality formalism incomparably sophisticated and powerful than the one we had previously in the transcendent context for the only topological varieties (and local systems hereunto), and satisfying even the formalism which I had succeeded in consistently duality.

My consistent duality works are exhibited in the well known seminar of R. Hartshorne "Residues and Duality" (only published in 1966) 585 (\*\*), those

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on the duality spreads in one or two chapters SGA  
p. 941

4, and especially in the seminary SGA 5, which was entirely devoted to it. And it is at time of writing only lines that I realize, suddenly, that apart from a few sporadic-precursors texts (in Cartan and Bourbaki seminars fifties), there is no systematic text **published**: and my pen, exposing the formalism and yoga duality, either in coherent context, or the context spreads. The presentations SGA 4 on this theme, centered around the single "duality theorem global "to separate morphism of finite type (establishing that  $R_!$ ,  $R_!$  are added), were written

584 (\*) Of course, I had already realized that in the case of an open immersion  $f: X \hookrightarrow Y$ , where the functor  $f^!$  coincides with the functor  $Lf^*$  of "restriction  $X$ ", it admitted of **not** (in the context of quasi-coherent sheaves) Assistant left. The assistant left  $Rf_*$  usual  $!$  ("zero extension out of  $X$ ") does not retain the quasi-coherent. On the other hand, I also verified that apart from quasi-coherent assumptions and even clean morphism basic point, there is no "duality theorem". Thus, the impossibility of defining an  $R_!$  under general assumptions me seemed sure and in the nature of things.

This is Deligne which was found in 1965 or 66 (just landed!) That one could make sense of  $Rf_!$  and recover Theorem consistent duality for a separate morphism not own finite type, provided to work with coefficients which are (complex) **pro-beam** quasicohérents. This beautiful idea has not yet had the fortune that could have been wait - no more than the original formalism consistent duality, it allowed to perfect.

Deligne has successfully taken this idea in his essay of a building "coefficients Rham" in the diagrams algebraic characteristic zero, promising test it nevertheless dropped to profit and loss as soon after my departure 1970. It is Mebkhout six years later, it was reserved to identify "the" good class "coefficient Rham" (Crystalline) I anticipated for ten years then. . .

585 (\*\*) The seminar in question (published in Lecture Notes in Mathematics, n° 20, Springer Verlag) describes most of my ideas on the formalism of coherent duality, centered on the formalism of the six operations, bidualité, and theory "Residual complex" (which are canonical injective representatives complex dualisants). These ideas were taken up in the analytical framework by Verdier and especially by Ramis and Ruyet. The seminar Hartshorne does not, by cons, various finest developments closely linked to this formalism: a theory of residues (for finitely patterns and dishes on any base), and a cohomological theory of different, which have never been published not (to my knowledge). I also developed in the 50s formalism of "determining unit" perfect complex, which was eventually be included in SGA 7 and whose editor (following the example already set by some "editors" SGA 5) has withdrew, after two years.

Finally, I note that in the wake of my thoughts coherent duality fifties, I had been led to so introduce and develop so slightly purely algebraic version of the **cohomology of Hodge** and that of **De Rham**, and particular formalism cohomology classes associated with an algebraic cycle (assumed smooth at first), and a theory of Chern classes, on the model of the one I had developed theory of Chow.

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by Deligne two or three years after the seminar, from my handwritten notes 586 (\*). As for the seminar SGA 5, it was practically sequestered for eleven years by cohomologistes my students to be finally published ( **after** the text-Kick Saw Deligne 1977) generously and unrecognizable looted, ransacked by care of "editor" -SiC Illusie, the entire devotion of his friend prestigious 587 (\*\*). That, in this ruin what was

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one of the best seminars I have developed and with USG 4, the most crucial of all p. 942 in my work surveyor - this is the only written record of my hand, or at least according to notes my hand, which evokes so slightly formalism and yoga stretches of duality, and, beyond this yoga yet partial, and irresistibly suggested by him, one of the six operations. My students took care to erase any trace of the latter yoga 588 (\*), suggestive of exceptional strength that had inspired my work on cohomology throughout the sixties. This was truly the "nerve" in the thrust "types coefficients " 589 (\*\*), the yoga patterns is the soul...

Such an absurd situation where an important advance in science, embodying in a vision nouvelle is eradicated by the care of the very people who were the first beneficiaries and custodians, could not be established without this other situation, she also highly exceptional, created by my departure subit and the conditions that surrounded. Also, take the turn that went events was prepared even before my departure and throughout the sixties by the situation of division where I found, cornered one hand by endless tasks of foundations that I was the only one able or vou-

dormouse assume 590 (\*), and secondly constantly approached with questions on topics often far 586 (\*) This wording Deligne is placed **after** the seminar SGA 5. Besides Deligne did not follow my notes to the letter, but variant of my method, Verdier was introduced in the context of locally compact spaces in 1965 (resuming essentially the model spreads). At that time there was ambiguity in anyone's mind about the paternity of all main ideas in duality, let alone on the paternity of duality spreads; it would have occurred to anyone (not even Deligne surely!) That to follow a variance of my original method may over the next two decades be used to fish in troubled waters, and assign Verdier duality spreads (Deligne while pocketing the rest of the "package" étale. . . ).

587 (\*\*) See the note about "The four maneuvers" (n° 169 (ii)), and sub-notes following the.

588 (\*) (May 8) I just instantly browse my handwritten notes for the first three presentations SGA 5 rating has quillusie kindly return me last year to my request. (He is the only ex-editors who took the trouble to return my notes I had entrusted to them ...) The first set consisted of a comprehensive "overview" of what had been accomplished in the previous seminar SGA 4, regarding the cohomological formalism spread and its relationship to various other contexts. The

second exposed develops extensively formalism "abstract" six variances. There is essentially a form full, but still effort to identify compatibility between canonical isomorphisms. (This was a kind of task more technical, unnecessary at a time when I wanted above all to "pass" this duality yoga, which I felt all strength.) Needless to say there are traces in publishing-Illusie neither one nor the other exposed. I had come to believe that (monopolized by

the more technical aspects of the seminar) I probably failed to expose the unifying vision. In retrospect, and a year almost to the day after the discovery of the "massacre" of the seminar SGA 5, I seem to have pinpointed today what was the **nerve** even to this operation-killing. This is not the disappearance of such or such other presentations, annexed by Deligne, looted by Verdier, saved from disaster by Serre torn a "whole" harmonious, for the sole pleasure it seems, by a Illusie. But it is the soul and the same nerve of the seminar, the constant and pervasive thread all Throughout this vast work done by one - he qu<sup>o</sup>Illusie has sought to eradicate SGA 5 without leave (almost) no trace. The name "six operations" is absent from the seminar, as it is absent from the work of my students, who had to tacit agreement not to say those words in the very rare occasions when one or the other is still faced with the worker said the deceased, which (while he is deceased) though it should deceive. . .

589 (\*\*) The key idea, too, has been eradicated and forgotten by my cohomologistes students. This is one of the first who will be again imposed on me from the first retrospective made "fifteen years after" my work and its vicissitudes, in note "My orphans" (n ° 45). This note, whose name affects more just and deeper than I would have dreamed then, has been written yet even before the discovery of the "Burial" (literally and strong sense of the term). The same key idea of the six

operations and "cohomological coefficient" comes across as almost a leitmotif when thinking in Crops and sowing denies reconnects with the fate of my work by those who were my students. See in particular the notes "The melody in the tomb - or sufficiency" (developing so slightly the "melody" or theme variations, types of coefficients), and "The tour of building sites - or tools and vision" (Notes n ° s 167.178).

590 (\*) I remember it working foundations of vast dimensions ended abruptly and until today, from the day of my departure. This is an eloquent sign of this "misunderstanding" I talk about in the note "The Nest Egg" (n ° 169 (v)). All

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the first foundations which absorbed me in the moment, and by the same token, often, more intensely and more directly fascinating <sup>591</sup> (\*\*). Rarely, even among the themes that I had let myself leisure deepen and develop (as that of duality), I found the leisure also write in proper form for publication cation the results of my work (in accordance with the high standards that are mine). Therefore often I was taken to let others (to whom I was totally confident, certainly) the task of drafting a (As was the case with the theme "duality" in both the coherent framework that discreet spreads), or develop such initial ideas I knew fertile (like the derived class, or coho-ogy crystal, to name but a few among many). In a "normal" circumstances, the a good faith response to the confidence that I was addressing myself to motivated students to learn my Contact their profession and major bases for their work to come, everything was for the best,

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and for more

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good of all concerned, including the scientific community. But it is true that this situation unusual put their hands (without ever before last year the idea has touched me...), and especially after I left, a **power** considerable. Immediately after my departure (if not even before...) Some among them were quick to abuse this power, to retract the work and vision, run down the worker, and take advantage of such tools fashioned by him they felt they use.

My consistent duality works have never enjoyed great popularity, it seems <sup>592</sup> (\*). On the other hand, those of duality spreads attract immediate attention. But it would be more accurate to say I believe that what attracts attention is that someone has "managed", no matter how, to demonstrate in the context spread the analogue of the Poincaré duality, one that was well known to all for almost a hundred years I imagine in the familiar context of oriented topological varieties. So this was a "good thing" for étale (there was little doubt that it was "good" for the Weil conjectures ( "a difficulty proverbial "...). This means that the mathematical public abreast of the famous conjecture, reacted in "consumer", which is reluctant to recognize a new and profound vision and assimilate, and retains a "result" to look familiar. More than twenty years later, I find that this vision of strength six types of operations and coefficients, speaking in a formalism amazingly simple, remains unknown to all (with the exception of the lone worker), when it is the object (when someone dares to make some allusion y) rigouillards ironic comments or <sup>593</sup> (\*\*). Such scattered ingredients my range are used here and there without referring to my person (and fathers spare any found), and especially the bidualité formality, since the great rush on the intersection cohomology after memorable Symposium (in 1981) that it will be question. But the

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vision , a childlike simplicity and

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perfect elegance, which nevertheless gave eloquent proof of his power <sup>594</sup> (\*), is ignored, the subject of the world was ready to bring her furniture and settle permanently in such houses I had built - but stir and handle trowel and plummet to build on and develop, and it was only under the pressure peremptory needs, there was nobody. . .

591 (\*\*) If I had listened to me, how often would I stand there the endless tasks of foundations that I coltinais service all to get into the unknown adventure that constantly calling me, true - instead of letting other fun to survey the new land I had discovered. Today I see that those lands are still virgins, or nearly need, and those in which I thought I saw the pioneers had since before I left already chosen to be comfortable annuitants. . .

592 (\*) As I pointed out in a previous note of b, p., This work has inspired those Verdier, Ramis, in theory Ruguet coherent analytic spaces. Clearly "always" (for me at least) that the same formalism can only Inside the rigid-analytical context (which, too, is still at the stage of childhood by the echoes that return to me).

Moreover, Mebkhout told me that the Japanese school analysis was quite inspired by "Residues and Duality" by refraining Besides ever name the worker. For these times, the opposite would have been surprising. . .

593 (\*\*) For details and comments, see sub-note "Unnecessary details", n ° 171 (v) including part (a) "packets thousand pages. . .".

594 (\*) For details about these "compelling evidence", see sub-note "Unnecessary details" (n ° 171 (v)), part (b) "The machines do nothing. . .".

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### 18.5. THE FOUR OPERATIONS (a corpse)

disdain for those who prefer to be despised (and plunder...), rather than understand.

If what I did with my hands and my heart has been ahead of its time in twenty years or maybe fifty, not by immaturity of the **mathematics** that I found by putting your hands dirty,

there is this thirty. It is the immaturity of men <sup>595</sup> (\*\*). And it is at that immaturity has faced my posthumous single student and successor, Zoghman Mebkhout. I was very fortunate, before leaving in 1970, are to be confronted in the form of misunderstanding, which never is departed provisions that remained friendly. Zoghman Mebkhout arrived on mathematical place more time than he recklessly continued the work, was entitled, he, after misunderstanding and disdain, and when the value of tool u **n** its results was finally recognized the malice of his elders and with all the weight of the unfairness of time but I anticipate. . .

One of the most important discoveries I have made in mathematics "and remains virtually ignored by all, was that of **the ubiquity** of the duality formalism that I had started to develop in the fifties: the "formalism of the six variances and bidualité" applies to both coefficients "Continuous" originally envisaged ( "coherent" theory), and "discrete" coefficients. This ubiquity is appeared as a surprise hardly believable in spring 1963 - thanks to it, and nothing else, I able to develop a duality formalism spread and reach what I call the "master" of the cohomology spreads. And from that time, I was intrigued, not dwell too it is true, by the issue of a theory would be "common", either in the schematic part, or complex analytic or even topological - a theory that "coifferait" both types of coefficients. The De Rham cohomology (an old friend me. . . ) Gave a first indication in this direction, suggesting to find a "common principle" in direction "modules integrable connection" (or "layered modules" maybe...). These give birth to a "De Rham cohomology" (discrete coefficients, morally), which is thus set

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in connection with the coherent cohomology. This approach later suggested to me the idea of "crystal" and <sup>p. 946</sup> of "crystalline cohomology" without sufficient yet (it seemed) to provide the key to the description a complete formalism of the six variances for the types of "coefficients", which in a suitable way, in-globeraient both discrete coefficients ( "Building"), and continuing coefficients <sup>596</sup> (\*). It seems not any of my students have been able to feel this problem <sup>597</sup> (\*\*)

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with the sole exception of Deligne. He devotes a page. <sup>947</sup>

595 (\*\*) For a primer thinking about it, see sub-note "Freedom..." (N ° 171 (vii)).

596 (\*) As I write this, I remember about it was still unclear. It was revived later, and I come back in a more detailed in sub-note "The absurd questions" (n ° 171 (vi)).

597 (\*\*) I had mentioned this problem to Verdier, after he developed (as I had suggested) the duality theory topological spaces (or at least a theory of embryo), on the model of the one I had developed in the context spreads (see on this subject the subnotes n ° s 81, 81). This was to be the mid-sixties. Obviously it did not "tilt" then - the same sense of the question (a bit vague perhaps, it is true) seems to have escaped him. Yet surely I had mention the De Rham cohomology, as differentiable complex analytical, which puts relationships and Serre duality Poincaré duality, on either type of coefficients.

(May 14) Moreover, from the fifties I knew we can generalize the duality theorem of Serre in the case of a com-

complex of differential operators between locally free sheaves onto a clean and smooth scheme, so as to encompass also De Rham cohomology (so morally cohomology with discrete coefficients). So this is a result of duality very close to that of Mebkhout in the analytical framework, which will be discussed in the following note. I had not pursued then this path, especially, I think, because I could not see how to make a "derived class" suitable with complex differential operators, for lack of a good concept of "quasi-isomorphism." It is also true that isolation where I worked on issues (coherent cohomology) that visibly interested nobody else in the world than me, was little incentive to cram additional generalization (with differential operators replaced morphisms linear) over those I had already emerged in my corner, in previous years. I was ready yet from the perspective of Mebkhout where passage to the corresponding 3-modules (the components of a complex of differential operators) gives a key to a perfect simplicity, to build the derived class needed. In 1966 Moreover, (but without realizing it so clearly) in my hands a dual perspective, that would have allowed me to do a class derived sheaves "pro-laminated modules" (an idea later developed by Deligne in his draft of a theory coefficients De Rham, which it will be questioned). Indeed, by associating with coherent pro-Module Module 739

seminar a year (with the IHES, in 1969/70 I seem to remember) to develop a formalism which allows him at least for one type of scheme  $X$  over a characteristic zero  $k$ , describe cohomology spaces (called "De Rham") which, in the case where  $k = \mathbb{C}$ , restore the "cohomology Betti ordinary complex (defined by transcendental way). The coefficients he worked with were of "layered promodules" and complex such promodules. It was not clear yet if these coefficients would fit into a formalism of the six operations 598 (\*), and Deligne has declined to pursue this path. If I remember correctly, that was missing above (\*) to give confidence, it was a description in terms purely algebraic (with blows of coherent modules or procohérents and laminations), valid so on any base of characteristic zero, the category of  $\mathbb{C}$ -vector bundles "algebraically Building "X- 599 (\*\*), which is defined by transcendental path when the base body is the body  $\mathbb{C}$  complexes.

#### 18.5.4.2. b. The work. . .

**Rating** 171 (ii) The work of Mebkhout beginning in 1972, are placed in the transcendental context (And technically difficult) analytic spaces. It is in almost complete isolation that become familiar over the following years with my work on cohomology and formalism derived categories 600 (\*\*\*), left behind by those who were my students.

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A conductive wire which

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gradually takes a leading role in reflection, is a striking parallel between duality continuous and discrete duality. The latter had meanwhile called "Poincaré duality-Verdier" without provided that person in the big world (and certainly not the new "father" Verdier) do not mine question a deeper reason for this parallelism. It is the reign of view "utility" and short view, merely using the tools already ready I had created, without asking any questions - especially no vague question, if not preposterous question is not mentioned in any published text, not even (and I realize that I am here to blame...) in those of my pen 601 (\*).

its **principal parts of infinite order**, which is provided with a canonical stratification or associated with a complex of operators

a complex differential laminates such promodules whose crystalline hypercohomology is identified with the hypercohomology zariskienne the envisaged complex differential operators. (See my presentations "Crystals and the De Rham Cohomology of schemes" (Notes by I. Coates & O. Jussila, in ten papers on the cohomology diagrams (p 306- 358), North Holland. - especially by 6.) the notion we can define the "quasi-isomorphism" to a morphism (differential) between complex of differential operators, as usual, in terms of the associated complex laminates promodules.

598 (\*) Again, my memory was fuzzy, and there is an error - it was clear a priori here for heuristic reasons transcendent nature dante, it **should** be a formality six operations. (See, for details, sub-note "... And hinders" n ° 171 (viii).)

My mistake was due to a deliberate visibly (flower of consciousness) to rationalize, to make sense of something that might seem inexplicable, namely the abandonment by Deligne a search direction "safe" and rich in promise. The reason indeed is not mathematical in nature!

599 (\*\*) I recall that this notion of constructability was introduced by me, among many variations (algebraic, real analytic etc.) from the fifties, at a time also when I was strictly only one interested in these Questions. (See my comments last year, in the sub-grade n ° 46 3 .)

600 (\*\*\*) (May 14) said Mebkhout me since those first readings of the mathematical literature, around 1972, were works of Japanese authors of Sato School. He struggled, he told me, to navigate, it seemed terribly complicated. There he found a reference to the book of Hartshorne "Residues and Duality", the reading was for him true relaxation. It is true that this book is beautifully written! The few words of introduction that I had written for this book, citing the ubiquity of formalism that is developed there, have inspired many. It is from there that he began to become familiar with my work, which is later became his main source of inspiration. In all these works and

exposed, he takes care to state clearly that source.

601 (\*) (May 14) Yet I remember that during the seminar SGA 5, I constantly had in mind the ubiquity of formalism I was developing, and I did not miss an opportunity to point out the possible variants in such other contexts, 740

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The same formulation of common formalism is an essential use of derived categories. mebkhout in is his constant work tool against the fashion wind and scorn of his elders, starting with one that (it is unclear if it is willingly or unwillingly...) is then shown in "father" of those categories, always know Verdier. Compared to the arsenal that I had introduced the new ingredient essential Mebkhout is microlocal analysis Sato and his school. Specifically, Mebkhout borrows them concept of three-module on an analytical complex manifold (equivalent to the concept of "modules crystal" I was introduced to 1965-66, which has meaning in wider contexts, including singular varieties), and especially the concept of 3-consistency and delicate condition holonomy on a coherent 3-Module. Moreover, it is an essential use of a Kashiwara theorem of 1975, that cohomology sheaves of differential operators complex associated 3-Module 0

holonomic are p. 949

Plot analytically. It was a point of view and the results I had no idea before Mebkhout not talk to me about it two years ago, and Deligne was ignored as much in 1969-1970, when his thoughts then were not followed, to a formality coefficients De Rham. This is **putting together the two currents of ideas** that Mebkhout reaches a common understanding of the two types coefficients of a complex analytic manifold smooth X, in terms of complex differential operators tial, or (and more specifically, in the most flexible language 3-Modules) in terms of complexes 3-coherent cohomology modules 602 (\*). This is his great contribution to contemporary mathematics. More specifically, if X is a smooth complex analytic space, designate by Cris \* coh (X) sub

full of the category derivative  $D^*(X, \mathbb{C})$  formed complexes  $\mathbb{C}$ -modules cohomology  $\mathbb{C}$ -consistent by  $\text{Cons}^*(X, \mathbb{C})$  the full subcategory of the category derivative  $D^*(X, \mathbb{C})$  formed C-vector bundles on X complex to analytically constructible cohomology, and finally

$\text{Coh}^*(X) = D^*$

$\text{coh}(X, Y, X)$  Full subcategory of the category derivative  $D^*(X, Y, X)$  formed complex  $O_X$ -modules coherently cohomology. Mebkhout highlights fundamental functions

$\text{Cons}^*$

$\text{coh}(X, \mathbb{C})$

u

M

((

PPPPPPPPPPPP

$\text{Coh}^*(X)$

NOT

xxpppppppppppp

(Meb)

$\text{Cris}^*(X)$

where the right functor N is the functor "circular", derived functor total extension of scalars the obvious inclusion  $O_X \rightarrow \mathbb{C}$ . The functor left M, or "**functor Mebkhout**" is likely much deeper 603 (\*\*). It is **fully faithful**, and its essential image is the full subcategory  $\text{Cris}^*$

$\text{coh}$  formed complex of  $\mathbb{C}$ -modules beam not only coherent cohomology, but more "holonomic" and "regular". These are subtle local conditions, first introduced by Sato School, the second ad hoc defined

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by Mebkhout 604 (\*), based mainly (said he) my p. 950

comparison theorem between algebraic cohomology De Rham cohomology and analytical De Rham for ideas and techniques that I developed as part of the discrete étale. It just seems incredible that I have not mentioned during the oral seminar, the problem of a synthesis of two types of factors, not least in the final presentation on open problems, also disappeared body and well-publishing massacre. Needless to say that no mention to such a problem is in this edition, carefully purged of anything that does not fit with the label rigor "volume technical digressions." . .

(May 19) See also in this sub-note "The dead pages" (n ° 171 (xii)).

602 (\*) For details about the language of 3-Modules, his relationship to the complex differential operators and the



crystals, see sub-note "Five Photos (3-modules and crystals)," n ° 171 (ix), part (a).

603 (\*\*) For a description "explicit" a closely related functor  $M^\infty$ , in the context of  $3^\infty$ -modules, see sub Note cited above n ° 171 (ix), part (b); "The formula of God."

604 (\*) The "regular" name is taken, of course, the classical language for "regular critical points" equations Differential functions of a complex variable. If  $i: U \hookrightarrow X$  is the inclusion of complementary  $U = X - Y$  of a 741

complex (ie the cohomology complex Betti). These conditions (and this is what for me is their main interest) are in fact "purely algebraic", keeping a particular direction if  $X$  would be replaced by a finitely scheme (smooth if you like, but it's not necessary) on a characteristic zero any.

The functor Mebkhout million (or "functor of God" 605 (\*\*)) describes itself as virtually the opposite functor functor

$$m: \text{Cris}^*(X)_{\text{hol.rég.}} \rightarrow \text{Cons}^*(X, C)$$

defined by

$$m F \mapsto \rightarrow CD(F)$$

dfn

$$= \text{Rhom}_3(O_X, F),$$

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restriction of the functor (defined on  $\text{Cris}^*$

$\text{coh}(X)$  as a whole) associating with each complex of  $3^X$ -modules (to

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coherent cohomology) of the complex differential operators (or "De Rham complex") associated with 606 (\*).

The constructability Kashiwara theorem implies that when  $F$  is holonomic (and a fortiori, when regular holonomic),  $DR(F)$  is in  $\text{Cons}^*(X, C)$ , which allows to define the functor  $m$  - a definition

Although obvious, childish, and yet that person apart Mebkhout (and until the "great

rush "still, five years later ...") had thought 607 (\*\*) (He would have required that recalls a

some yoga, the derived classes, everyone with one accord decided to bury,

alongside the deceased who had introduced among other bombinages the same style ... 608 (\*\*\*)).

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In addition,

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divisor  $Y$  in  $X$ , regularity as defined Mebkhout (for a  $3$ -Modules  $C^X$  complex), "along  $Y$ ", is written in saying that the morphism

$Ri_{\text{mér}}$

$$* (C^U) \rightarrow R^*(C^U)$$

of the "direct image meromorphic" of the restriction  $C^U \subset C^X$ , to the ordinary direct image, induces a quasi-isomorphism for complex De Rham associated.

In the case where  $F^U$  is reduced to a "local system" ie a beam  $O_U$ -consistent integrable connection, this concept equivalent that of Deligne. This is clearly inspired, too, my comparison theorem (with the difference that

Deligne has custody report, while Mebkhout constantly takes care to clear his sources). mebkhout

only became aware of the notion of Deligne after having introduced its own definition, which is transcendent nature. he

had not previously sought a purely algebraic description of his condition. The Working Deligne showed that in the

particular case under consideration, the algebraic condition Deligne implied that of Mebkhout Mebkhout and verifies that the reverse is

also true. This therefore provides the key to a purely algebraic description of the regularity condition Mebkhout for every  $3$ -modules of complex coherent and holonomic cohomology.

Mebkhout told me that the Japanese have a concept of "micro-differential system with regular singularities", they used in a completely different spirit (for analytical purposes, not geometry). After the rush on the right of the theorem

God, this was a ready means (among many others) to muddy the waters and to retract the pioneering work

of Mebkhout. It seems that the two notions are equivalent - and it's likely, given the deliberate mess in the state

about, that nobody has ever bothered to check. Mebkhout never worked with the concept of such regularity

he had introduced in 1976 (and which appears in his thesis, submitted two years later).

605 (\*\*) For the origin and meaning of the name "theorem (or functor) of God", see note "The unknown service and theorem

God" (n ° 68) also written before that I have knowledge of the hoax Symposium Pervert, nor yet

the "Burial in all its splendor."

606 (\*) See note on this subject already quoted "The five photos (crystals and 3-Modules)" n ° 171 (ix), part (a), "The album" coefficients De Rham "".

607 (\*\*) (May 7) It is necessary to call the **two** functors  $m, M$ , setting in one direction and the other equivalence classes

crucial, the **functors Mebkhout**, and even for the functors  $m^\infty, M^\infty$  related to  $3^\infty$ -modules. (Regarding the latter,

see note cited "The five pictures" (n ° 171 (ix), part (b).) In composing these functors with functors natural dualisants it

Two other quasi-inverse functor pairs from each other,  $(\delta, \Delta)$  and  $(\delta^\infty, \Delta^\infty)$ , contravariant them, and more convenient

in some respects (see note cited). These are the four " **contrafoncteurs of Meckbhout** ".

608 (\*\*\*) (7 May) More than once Meckbhout has been treated as a funny, which believes that writing arrows between derived categories (Asked for a little!) And Rohm is doing math. . . He did not shake left provided, any more than I in time when I introduced (in 1955) the Ex <sub>i</sub> global and local modules bundles (pending with Rohm or without underlined), which gave seasick at all justified and the most express reservations to me (at least 742

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condition of **regularity**, beyond that holonomy, was released by Meckbhout "on measures", such so precisely that it becomes reasonable to expect that the functor  $m$  and  $\text{small}$ , is fully faithful and even an **equivalence of categories**. He arrives at this conviction in 1976. He eventually prove, under a form very close at least 609 (\*), in his thesis in early 1978.

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It©mainly there **the** great new theorem provided by Meckbhout representing the culmination of eight per cent. 953 Years of persistent work, pursued in complete solitude. It contains, in one pithy statement, a whole range of profound results, increasing generality, patiently released and proved one by one, between 1972 and 1980. For some great milestones in this solitary journey to discover a "philosophy" in new the cohomology of varieties, I refer to the sub-note "The three milestones - or innocence" (n ° 171 (x)). In the this note, my remarks will mostly describe briefly the new panorama that has, at After this first step of the long labors of the solitary worker Zoghman Meckbhout.

The crucial fact (clearly recognized Meckbhout already in 1976) is that the category  $\text{Cons}^*(X, C)$  (from Nature "topological") can be interpreted, with the functor Meckbhout million, as a subclass full of Cris category \*

$\text{coh}(X, C)$ , which has meaning within the scope of Algebraic Geometry "abstract trafficking"; it may be interpreted as" morally "as a kind of" derived class "formed with complex differential operators in the ordinary sense 610 (\*) The full subcategory in question, defined by until 1957, the year of the Riemann-Roch-Grothendieck. . . ).

All this has not prevented Meckbhout to trust their own intuition, and follow where it led him. He started to work bare hands, without experience, without help from anyone. He was **sure** that he sensed the theorem must be true - all indications that he had in hand were concordant. With a little experience, it would have been obvious even he had any in hand to prove it, with the now standard means that the first of my students come to apply in a jiffy. But reduced to its own resources, the theorem seemed dizzyingly remote and inaccessible - is he barely dared hope he never demonstrate!

If indeed struggled to prove it, for nearly two years is that it did not have the advantage, as my students had been, to be supported by a caring elder, and get to my touch a certain standard of technical unscrewing of building beams, attached to the resolution of singularities to Hironaka. The statement that generated is a profound statement

certainly, and the proof is too deep, but standard in nature today. In retrospect, it appears that the difficulties he had to overcome was mostly psychological, rather than technical: working in counter, and fully reduced to his own lights. . .

609 (\*) (May 5) In his thesis, Meckbhout states and proves the equivalence theorem corresponding to  $3^\infty$ -modules, and gives remarkably explicit expression of the quasi-inverse functor  $M$ . On this subject the sub-171 score (ix) (part (b)), and also sub-note "Hatching a vision - or the intruder" (n ° 171 <sub>1</sub>). Meckbhout had succeeded in 1976 in the belief that the two functors  $m$ ,  $m^\infty$  (thus also the function  $i$  expansion scalar, referred to in the last sub-note quoted) are equivalencies, and the explicit form of the quasi-inverse functor  $m^\infty$ . The result contained in his thesis concerning  $m^\infty$ , of 1978. From that moment, he hands all the ingredients for the demonstration (analogous, but with difficulties Additional technical) in the case of  $m$ .

View the general indifference that greeted his thesis, passed in February 1979, it does not then attempt to write a demonstration in shape for the case of  $m$  as well. The ingredients are the same as for  $m^\infty$ , and were inspired demonstration My comparison theorem for the De Rham cohomology of complex algebraic varieties (of which he had taken knowledge in 1975), and SGA 5 Breakaway techniques (he learned in "good reference" Verdier, while Seminar 5 USG continued to be carefully sequestered by the care of my dear cohomologistes students). Only end 1980, given the importance that took his ideas for the demonstration of the conjecture Kazhdan-Lusztig, he takes the trouble write a detailed demonstration in the case of  $m$  (which is not available in advance of a quasi-inverse functor). This demonstration is published in "Another equivalence classes", *Compositio Mathematica* 51 (1984), p. 63-88 (manuscript received on 10.06.81).

I stress about that between 1975 and 1980 (apart from a hint of a few lines of Kashiwara in 1980, which will be issue in the sub-note "The Mafia" n ° 171 <sub>2</sub>), nowhere in the literature outside only work Zoghman Meckbhout, there is no question of the functor  $m$  or  $m^\infty$  nor a "philosophy" of duality, connecting precise discrete coefficients Building analytically, and complexes of 3-regular holonomic modules, or complex of  $3^\infty$ -modules holonomic. As we shall see, when at last the importance of this relationship is recognized with "Kazhdan-Lusztig" and rush on the cohomology

of intersection (under the guidance of Deligne), the name of Zoghman Mebkhout is removed without fanfare by agreement hushed, smiling and discreet, and a ruthless efficiency. . .

610 (\*) For the precise relationship between the two points of view, I refer to the sub-grade widely cited "The five pictures" (n ° 171 (ix)), part (a).

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holonomy of Condition (Sato) and regularity (at Mebkhout) is obviously **the** right category of "Coefficients Rham" I planned since the sixties, and still missing in my range in characteristic zero, to complete and to interconnect as a single large fan, the "coefficient l-adic" I was released in 1963; it is also the category Deligne had tried to enter the late sixties, but there (it seemed) in a way that satisfies him. This class, obviously, will have a key role to play in algebraic geometry (particularly in the description of the category patterns on a basic scheme  $X$ . . .). The name that is required for this category, for me at least, is that of "**class coefficients De Rham - Mebkhout**" <sup>611 (\*\*)</sup>, denoted  $\text{DRM}^*(X)$  (or  $\text{Meb}^*(X)$ ), or  $\text{DRM}^*(X/k)$  (or  $\text{Meb}^*(X/k)$ )

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in the schematic part, when  $X$  is a scheme of finite type over a field

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$k$  characteristic zero <sup>612 (\*)</sup>.

It is via the functors diagram (Meb) above, which summarizes the philosophy of Mebkhout (dating back to 1976 and established by him in subsequent years), the **consistent crystalline coefficients** (ie objects  $\text{Cris}^*$

$\text{coh}(X)$ ) may be regarded as a "common generalization" "discrete" coefficients (Building) and "continuous" (consistent). The class formed by the first is identified in all cases by Mebkhout functor  $M$  (a functor profound nature), to the **full subcategory** of the category crystal-consistent line formed coefficients De Rham-Mebkhout. The situation is worse for the functor tautological  $N$ , which is not fully faithful. But to comfort us and to complete the picture, we may be added that in each of the categories in the presence, there is a **functor dualizing** natural Don-  
ing rise to bidualité theorem ("trivial"  $O$  for  $x$ -modules  $3$  and  $x$ -modules, and using the strength the resolution of Hironaka singularities in the case of vector Building C-beams) on the model I had cleared the coherent framework (commutative) First, in the discreet then spread (in 1963) <sup>613 (\*\*)</sup>. That said, the two functors

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$M$  and  $N$  are compatible with natural functors dualisants <sup>614 (\*)</sup>.

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611 (\*\*)  
The general misunderstanding of the critical role and significance of this category already well reflected in the fact that it has not received name or terse notation. Instead (in texts that I looked) authors merely to vague references to "regular holonomic differential systems" (well that end it find!), "construction" or "Correspondence" or "relationship" (supposedly well known) between them and beams (E-buildable - and still, is it necessary to say, strictly ignoring the one who was the lone craftsman, setting in motion all the great hype the new cream pie beautiful people, "the" 3-Modules ).

612 (\*)  
In the algebraic case, we must impose, in addition to the condition of "regularity" Local, a condition of regularity "to infinity" (In the case of a non-own variety) to find the "good" coefficients De Rham - Mebkhout that will match, if the base body is a complex body, the C-vector complex  $X_{\text{year}}$  to cohomology sheaves **algebraically** (not just analytically) Plot For these coefficients as we have a "theorem comparison", generalizing my result on the De Rham cohomology, namely that the "total crystalline cohomology" gamma-ray

cries ,  
taking the algebraic point of view (zariskierien) or transcendental sense, is "the same". This statement in turn must be considered

as a special case of a more complete statement, namely that the "six operations" in algebraic terms are "consistent" with operations in six transcendent perspective.

If my students were not so busy burying the master©work, it is the very beginning of the seventy years (if this is from the sixties. . .) They would have cleared the theory coefficients that was necessary, in all its simplicity and all his power. . .

613 (\*\*)  
(5 May) The extension, context spread to the analytical context of my results bidualité, and stability of the constructive bility by the Rohm operation is automatic and I also was known as early as 1963, Verdier was working with me for three years, putting himself in the hot yoga derived categories (which he had undertaken to make systematic theory) and consistent duality. This is my mouth that he learned techniques that extend the consistent duality formalism if discrete coefficients. As we have seen, it is appropriate yoga duality and bidualité in the context analyzed complex tick in "good reference" thirteen years later (in 1976), with the connivance of Deligne and my classmates cohomologistes all well aware of the situation.

In the edition-killing SGA 5 the following year (1977), Illusie retained (in the statement I) bidualité theorem,

so that for a reader of texts, the trickery Verdier is obvious - but apparently it was considered as normal by all (seen these days ...). By cons, Illusie abstained to include the results of stability constructability by Rohm, I had of course given **prior** even to articulate and demonstrate bidualité theorem, which my demonstration (copied by Verdier) does not depend. So (you still have to!) Illusie merely establish stability in question when the second argument is complex Dualising! ! ! This was a way to cover his friend

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Moreover, if  $F, F$  are crystalline coefficients duality on  $X$  Mebkhout proves that complex C-vector "crystalline cohomology"  $F$  and  $F X_{615}$  (\*\*)

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p. 956

Gamma-ray  $\text{cries} (F) !$ , Gamma-ray  $\text{cry} (F)$

as complex topological vector spaces, are "duality" by natural mating in other words we have a coupling **which is a duality** ( $d \text{ @VT}$ )

$H_i$   
 $\text{cries} (X, F) ! xH_{-i}$   
 $\text{cries} (X, F) \rightarrow C$

(For any integer  $i$ ). This duality theorem "cap" duality ("absolute") known in the case of coefficients discrete (that Mebkhout called "Poincaré duality-Verdier"), and in the case of coherent coefficients (that Mebkhout called "Serre duality"), in a duality that I would call "duality Mebkhout" and that it has called "Poincaré duality-Serre-Verdier"  $_{616}$  (\*).

Verdier, making a little less apparent that from beginning to end (and nearly three pages of which was discussed in its place) Article Verdier is copied over my exposed SGA 5. The best part is that the stability in question is already a corollary the immediate bidualité formalism (which prevents it is mathematically crazy to pretend not establish Stability constructability by  $\text{Rhom} (F, G)$  that when  $G$  is the complex dualizing). But the complacent Illusie refrains mention this corollary in his presentation, in order to keep the appearance that the result of stability that appears in "The good Reference" buddy would indeed of his own.

One may wonder why, under these conditions, Illusie has still kept theorem bidualité - for massacre massacre, there was more to it close though! But if he had emptied, he would have had the shot as empty uphill Formula Lefschetz-Verdier (making it an essential use) - that is to say, precisely the "Trojan horse head": the formula which supposedly crucial role in SGA 5 should justify the brazen operation "suddenly saw" his other buddy, making break the unity of my work on the étale.

Refélicitations my ex-student Illusie, the clever "editor" -fossoyeur. . .

$_{614}$  (\*) For the tautological functor  $N$ , this compatibility is itself tautological. By cons, for the functor  $M$  Mebkhout (or, what amounts to the same, for its quasi-inverse  $m = (L \mapsto \rightarrow DR (G) = \text{Rhom}_D (O_X, G))$ ), this is a deep result, proved by Mebkhout in 1976 (under the name of "local duality theorem"), along with the global duality theorem for 3-modules, of which it will be question time. This prevents "everyone" now uses this result for granted, especially (something that goes even more so) without the slightest allusion to some unknown wave. . .

$_{615}$  (\*\*\*) I remember (see "The five pictures",  $n \circ 171$  (ix)) that the crystalline cohomology ("absolute") of  $F$  on  $X$  is defined as Gamma-ray  $\text{cry} (F)$

dfn  
 $\text{Rhom} =_D (O_X, F) \simeq \text{gamma-ray} (\text{Rhom}_D (O_X, F) = \text{gamma-ray} (CD (F))).$

On the other hand, the index! refers cohomology (crystalline in this case) to clean surfaces, ie

Gamma-ray  $! (F)$   
dfn  
 $= \text{Gamma-ray} ! \text{Rhom}_D (O_X, F).$

$_{616}$  (\*) As I said already elsewhere (in the note "The gossip",  $n \circ 63$  ), Mebkhout" could do no less "than do stunts hat to his "benefactor" Verdier (since it had communicated to him the providential "good reference") everywhere where he had the chance. Yet **none** of the essential ideas for any duality (and even less if it were, to which the cap) are due to Verdier. In fact, apart from the duality theorem of Poincaré and Serre in their form Initial, which certainly served me as a starting point, all the main ideas contained in the formalism of the six variances and bidualité I introduced and developed at length in both contexts, consistent and discreet, in solitude. It is with this in mind that I wrote last year in the note "The victim - or both silences" ( $n \circ 78$  @as the "protectors" of Mebkhout "had been willing he carries his hands a small corner of the coffin with my body." It was just that I also remember at this point that Zoghman had the courage, when he felt well yet what wind in the beautiful world, to repeat clearly in each of his articles he was inspired by my ideas, instead of doing like everyone else and to plunder the deceased while passing over in silence (in writing), and displaying condescending air (in words). As for the name "Serre duality" that ended up giving the coherent duality theory that I had developed over years in total solitude, it is all the more salt (and Serre, who do not ask for much, the best yet appreciate that person!), that Greenhouse had shown a total disregard for my duality work, depriving me of the single contact

I could have hoped for my cogitations! I think I can say also that this lack of interest is kept intact until today, including the notion of derived class (and other unnecessary details...).

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These, in my vision, the first steps of a vast dimensions duality program including in particular (among other (171 (xi))) the development of a formalism of six operations (and biduality) for the coefficients De Rham - Mebkhout on type schemes over a field of characteristic zero (until something better). Given the conditions of isolation and the atmosphere of indifference where Mebkhout had La-vailler, he could be issue

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for him to develop a complete formality, as the one I had developed

p. 957

in both contexts he was inspired (171 (xii)). Among the main results he emerges and proves during the eight years 1972-1980 (171 (x)), the one that strikes me as the most important from the perspective of my program sixties, of course, is highlighting **the** right category coefficients crystal, called "De Rham - Mebkhout". It turns out that this is also the result that, from October 1980 experienced fortune brightest, even staggering, despite the fact it was appropriate (as before the l-adic cohomology, or pie crystal cream since. p) as a **tool** only, a torn vision that gives it meaning and its strength.

More so than other results Mebkhout and just like in my work developing the bidualité formalism and six operations, the language of derived categories is essential here to clear simple and profound relationship between discrete coefficients and coefficients consistent <sup>617</sup> (\*) described in the-orth of God (aka Mebkhout never named...). Thus, it is almost twenty years after the creation of the tool cohomology

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logic spreads (that everyone uses today for granted, while dealing with the

p. 958

disregard the vision that gave it birth. . . ), And thanks to this result (now "pie-in-the-cream") of an obscure student posthumously, that the language of derived categories will be rehabilitated suddenly (as if it had never been buried. . . ) Under the limelight and the cheers of the crowd came to cheer yesterday buriers playing (modestly) new fathers. But again I anticipate. . .

#### 18.5.4.3. c ... . and boon

**Rating** 171 (iii) It is Verdier which more or less figure "thesis supervisor" of Mebkhout whose work seven years had been in complete solitude. There was at no time interested in this work young man, visibly bounded as he was stubborn - a vague grothendieckien lingered that addresses the top its magnitude. Over the four years since the first meeting in 1975, it will grant three "interviews" in all and for all to this chap who comes from nowhere. None of my other students cohomologists <sup>617</sup> (\*) (May 7) Specifically, in 3-holonomic module (reduced complex at zero degrees) the functor of God associated in general a building complex of C-vector which has more than one nonzero cohomology beam and vice versa. The example simpler and striking is where one takes a divisor Y of X, from which an inclusion  $i: = X \setminus Y \hookrightarrow X$ , and the sub-beam  $i^*(W \cup )$  formed by the functions meromorphic along Y. It is a profound result Mebkhout obtained in 1976 (and absorbed then the theorem of God) that this is a regular holonomic and 3-Module (person before Mebkhout had never even thought to look at the beam as a 3-module, and to suspect it was more even that consistent. . . ). His transformed by the functor of God is  $R^*(C \cup )$ , which beams cohomology nonzero dimension 0 and 1 at less.

This is one aspect of the philosophy of Mebkhout who was absent from the approach of Deligne, which got a dictionary Building between C-vector beams and some of proobjects Coh ( $O_x$ ) (the category of coherent Modules  $O_x$ ) equipped with a stratification, without having to go to complex and derived categories. (It still took care to involve them, at a time when I was still in the area and where the idea would come to bury a person that updating said categories. . . ). This is (at first glance at least) a benefit of the approach of Deligne, closer to intuition direct geometric discrete factors - but it is also a sign, perhaps, that his approach is shallower. I have tend to believe that it will still have a role to play, though, but in "tandem" probably with the view of Mebkhout which (I presume) is somehow dual.

(May 24) For details on this, see the sub-note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), part (C), especially p. 1009 and following.

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deign not be interested in working said chap. Its scope for their own research eludes them completely (as it blindingly obvious, even to a crumbling like me who "hook" of it

for fifteen years. . . ). They are too caught up in their trip-funeral, and a dull routine turn-crank, to be able to understand something new without presenting business card and without primers, with the only force things simple and too obvious. For a long time they buried their own creative abilities, merely be consumers of fashionable branded products. Thereafter, they will nevertheless largely take their revenge on the intruder who is allowed to see what their had escaped to them as to all (although they had any yet, as he and beyond, to see and make. . . ). But again I anticipate. . .

The defense takes place on 15 February 1979 in the general indifference. Mebkhout sends his thesis to all mathematicians he could think, rightly or wrongly, that they were interested in the cohomology of varieties analytical or algebraic - starting, of course, for all my students. Among those who received a copy of his thesis, **no one** will only acknowledge receipt of the item, or send a word of thanks is lying. It is true that the theory of Mebkhout feels even more (it seemed) such of its articles, conditions of adversity that had surrounded - she appeared thick and not easy, to say the least, and those who were not in on it had apologized for not hanging pronto. By

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against, I found p. 959

Oral explanations Mebkhout gave me his philosophy perfectly clear and immediately convincing, and there is no reason as he could give Verdier (1976), Berthelot (1978) Illusie (1978) and Deligne (1979) were less than those to which I was entitled.

This is the Bourbaki seminar June 1979 Deligne learns from the mouth of the Mebkhout " **correspondence Riemann - Hilbert** . "contained in the unread thesis (This was the name given by Mebkhout to equivalence category (or "dictionaries") which was discussed earlier.) Apparently, Verdier had never thought yet, over the last four years, to have a word with Deligne on the work of his dark student, whose work visibly interest completely escapes him until about the time of the Symposium Pervert in 1981 (where Deligne had to undertake to explain what it was ...). At Deligne against it by could only "to tilt" immediately - it was **the** solution, complete and concise, the problem he had himself left behind ten years before!

The reflex that seem self-evident in this situation (to the same point that I can not at this time yet, to imagine how we might act differently. . . ) Is immediately to congratulate the young unknown to have finally found the last word of a question my faith, deep, on which we had for a spine whole year, and we ended up dropping profits and losses. The mores have changed. . . Deligne, always benign course, is confined to a compliment wave (and yet it warms the heart to the candid Zoghman, not spoiled it must be said, well no idea of what awaited): yes, he had received his thesis and had even Read the introduction, and it was found that these were "beautiful mathematics." For Zoghman it was a day pomp! It was surely the first time (and last as well...), Where he is entitled to a compliment from so great man that everyone knows and quotes. . . 618 (\*) I do not know what happens in the head Deligne at that time and in the following year, on this remarkable theorem he had learned the mouth of a stranger. I assume he must ask around

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it 619 (\*) - Is it always the com- p. 960

618 (\*) (May 14) This is also the one and only time Mebkhout had the honor of a conversation with Deligne.

(June 7) For another compliment, from the previous year (June 1978) and the mouth of Illusie this time, see Note "Card white for looting - or the High Works "(n ° 171 4 ), especially 1091 page.

619 (\*) (14 May) On reflection, and from what I know also of Deligne, I doubt he really has "talked about him" before they do it with a clear idea and a definite plan. See note "Waltz of the fathers" (n ° 1764) about the game very particular played by Deligne, and the role it has to play two straw-grandfathers Beilinson and Bernstein. (See also "Market fool - or puppet theater, "notes n ° 172 2 (e))

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communicates in October the following year 620 (\*\*\*) to Soviet mathematicians Beilinson and Bernstein, guessing surely they will use. The same year, in fact, it is this "match" (always called "to Riemann-Hilbert "when we deign to name and without name Mebkhout is never uttered) that is the essential ingredient, the **new fact** that had previously failed to demonstrate a conjecture celebrates 621 (\*\*\*) I hardly know the name, "Guess Kazhdan-Lusztig". This is the shot Sending the same time, a sudden and dramatic revival in the cohomology of varieties algé-bricks, finally emerging from a long stagnation of more than ten years (if we exclude the work of Deligne Weil conjectures). This unexpected revival is reflected in the following year, the "happening" of Symposium Luminy June 1981 on "Analysis and topology of singular spaces" 622 (\*\*\*)).

#### 18.5.4.4. d. The day of glory

**Rating** 171 (iv) About This "Memorable Conference", I refer the reader to note "The Iniquity - or sense a return "(n ° 75) and the following notes, still hot and written in amazement (the word is not too strong) discovery. These notes form the procession VII of the burial, which I named "The Symposium

- or bundles Mebkhout and perversity. "

Suffice it to recall that in the Introduction to the Proceedings of the Symposium, signed by Bernard **Teissier** and Jean-Louis **Verdier**, the famous "correspondence Riemann-Hilbert" is presented as the "Deus

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p. 961

machina "of the Symposium. It is the same in the main section, which forms (with the Introduction cited) the Volume I of Acts, article by **AA Beilinson, J. Bernstein and P. Deligne** (and in fact written and presented at the Symposium, by the latter, in the absence of the other two co-authors). Moreover, the first two authors appointed were informed by care Mebkhout (regardless of Deligne) on

ins and outs of his theorem, from the previous year (November 1980) - Mebkhout had even purposely moved to Moscow for this purpose <sup>623</sup> (\*). Teissier also knew first hand and from

long - not talking about Verdier, who had chaired the Mebkhout thesis jury. . . Finally, I add that

had been decided "in extremis" to ask Mebkhout to make a presentation on the 3-modules theory (that nobody except him knew too, among the people there) Mebkhout took the opportunity to inform

Colloquium in full force <sup>624</sup> (\*\*) on the theorem that he modestly called the Riemann name and

<sup>620</sup> (\*\*) (May 14) This is apparent from a letter of Deligne to Mebkhout (received on 10 October 1980). For details of the episode Kazhdan-Lusztig, see sub-note "The Mafia" (n ° 171 2), part (d), "The General Repetition".

<sup>621</sup> (\*\*\*) The same conjecture is demonstrated independently, yet with a remarkable set at the same time (in

few days near) by Brylinski- Kashiwara, with the same main ingredient and the same manipulation retraction, and role-key to this development, and the name of the author of it below For more information, see sub-notes already quoted "The Mafia" (n ° 171 2)

parts (c) and (d).

<sup>622</sup> (\*\*\*\*) The Conference Proceedings are published in Asterisk n ° 100 (1982). These Acts are also printed in December 1983 and appeared in January 1984, nearly two years after the date marked on the volume.

<sup>623</sup> (\*) See, on this instructive episode, sub-note quoted "The Mafia" (n ° 171 2), part (d) "The General Repetition (before Apotheosis)".

<sup>624</sup> (\*\*) (May 14) Regarding the participants in this symposium strange, very "grothendieckiennes math festival", but with silence

the absolute ancestor deceased himself, as the dark posthumous student "who had the gift... to gather all

beautiful world "... As only students" pre-1970 "to participate in this symposium, there was Deligne and Verdier, but already sufficient

for well take center stage. Strangely, Berthelot and Illusie (whose work has been particularly marked,

I could tell by the lack of perspective Mebkhout unearthed there in large bands) are not the party. On the other hand,

Contou-Carrere (student "after") it is lost, all glad we had invited to tell his resolution method for

Schubert cycles.

I remember he came back euphoric, fully identified with all these brilliant and famous people he felt

to you and yours, and who came to listen, obviously interested but yes! He took contrite air to talk about Mebkhout

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Hilbert without leave open any ambiguity (one suspects) about the paternity of this result, which had the gift (unexpected for him as for all) to meet all these people.

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Moreover, it is in vain that the reader seek traces of the presentation Mebkhout in the Proceedings of the Symposium. p. 962

Verdier kindly explained afterwards that only items with results **new** would

included in Acts, while those of his thesis already dated from two years old. It is in vain as the drive seek trace in said Acts of any citation or any indication

so little specific about the origin of this famous theorem, which is however not due to Riemann nor

Hilbert. It will be difficult also to find traces of the name of Zoghman Mebkhout. This name does not appear in the first volume, either in the text or in the bibliography. In the second, it appears twice in the

bibliography by references- "go!" (One can not say that we have not mentioned!) Of the pen and Brylinski Malgrange - references that have nothing to do also with the theorem of God - aka Riemann

Hilbert - aka Deligne (and especially no Mebkhout) <sup>625</sup> (\*).

which was open to him bitterly, but he would not have said why too - for him Contou, in any case, clearly the Life was good!

That was in June 1981. Four months later, (in response to its unique application for a job in Perpignan) is the good slap thrust forward, hard cashed by him as a humiliation and an affront. (See, for this episode, the note "Coffin 3

- or jacobians too much on "n ° .. 95, pp 404-406 This particular note was written before I do it again

rapprochement with the episode involving Contou-Carrère, slightly dropped undoubtedly the brilliant Symposium.)

<sup>625</sup> (\*) (14 and 26 May) Other participants already mentioned, I read namely the participation of **Brylinski**, **Mal-**

**barn** and **Laumon**. All three were well aware of the work of Mebkhout who had the opportunity to inform each detailed manner, even outside the lecture he had given at the Symposium. That did not stop Bry-

Linski and Malgrange, in their article published in the Proceedings, which uses essential way ideas Mebkhout and Theorem of God, to retract both the crucial role played by the appearance of these new ideas and new tools, the name of their author.

As for Laumon, it will catch up later, in an article in collaboration with Katz. This is the same N. Katz who had distinguished already in 1973 with "SGA operation 7", which was mentioned in the note "Episodes of escalation" (n ° 169 (iii), Episode 2). It was also informed by Mebkhout its results in 1979 (see the note about "White Card looting", n ° 171 4). This is the article "Fourier transform and increase of exponential sums" (which is as the doctoral thesis of Laumon), which article circulates as preprint two years (I even got a by exemplary care Laumon). These authors develop a Fourier transform to the coefficients l-adic on the model that introduced by Malgrange in 1982 in the case of 3-modules (in the wake of the works of the unknown wave without mention of his name, as of course). The work of Mebkhout represent the foundation of heuristic theory developed by Malgrange as that of Laumon-Katz, just as they were to the article already cited Beilinson - Bernstein - Deligne (on said beams by them wrongly "perverse") That said, Laumon and Katz also follow the general movement (no mention of the unknown or service in the article, nor in the bibliography - any more than that mention of the ancestor, it goes without saying. . . ), Following the tone set by Deligne, Verdier Berthelot Illusie Teissier, Malgrange,

Brylinski, Kashiwara, Beilinson, Bernstein - I apologize for the alphabetical order in any case it already twelve directly and actively involved in the bright-deception scam Symposium Perverts - besides Hotta are putting hers Moreover Pacific, and thirteen!

Malgrange is also no longer quoted in the article in question - apparently there are allies authors cliques that cite them with a vengeance, avoiding mention those next door even when they suck on them to outdo. All ways when it comes to the ancestor or unknown wave, there they all agreed. It often brilliant math, surely - but I am old games, the mentality is not indifferent to me and it takes away the appetite to read, and in extreme cases, even to do.

Not those they do, anyway. The smell is too painful. . .

I also took a look at the article by JL Verdier, "Specialization and bundles moderate monodromy", published in those Acts. Unsurprisingly course, I saw the "Riemann-Hilbert correspondence" without referring (in the text or in the bibliography) to the unknown wave which he had chaired the thesis. He had forgotten, obviously ... It is also a matter Riemann-Roch theorem spreads (the name sounds familiar ...) - and I saw that too in the article Laumon-Katz. As neither one nor the other breathe a word of a deceased, I think that this "theorem" - there must have surely Messrs. Riemann and Roch, as this particular case is among the "technical digressions" and "non-sense" SGA 5 (not including the presentation of conjectures, providentially emptied by providing clever and "editor" Illusie...). Mebkhout had also foreseen in 1977 a link between philosophy and the Fourier transform at a time so where he was strictly only one interested to duality yoga, connecting 3-modules and discrete factors (such as myself the formerly had to formalism consistently duality and spreads). This intuition "Fourier transform" remained vague - the context was not then more encouraging for him to continue in this way, for me, in 1950, to expand my  
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Going back to the Conference in the flesh, we must believe that none of the brilliant mathematicians assembled p. 963

in these places, deigning to come and listen to the presentation made them an unknown wave service, no one noticed that "correspondence Riemann-Hilbert" that it presented to them as of his own, was well the same one that already so brilliantly introduced brightest of them, as the keystone heuristic his brilliant presentation, which formed (in the opinion of the organizers, Teissier and Verdier <sup>626</sup> (\*)) the "nail" everything brilliant Symposium on said beams (one wonders why) "perverse". Always is it that none of them was surprised, should we believe that the name of the unknown wave was not delivered in this paper, which flew so high that there was certainly no need to clutter up the few; or two years half later, with the publication of Acts (early 1984), the name of that unknown face or not, nor in the introduction (already mentioned), nor in the article Deligne et al. This item left Moreover little instead of a doubt about the true authorship of this corre

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dance, the lead author

p. 964

presenter and magician- <sup>627</sup> (\*), with his usual modesty, has also failed to appoint, not even the name of his two illustrious precursors. If that is yet who are surprised they are not done know until today - not to me, anyway, and certainly not the main question that provided the sauce for stuffing, namely the student posthumous unknown and rigorously as it should be, as now front - Zoghman Mebkhout <sup>628</sup> (\*\*).

#### a1. Unnecessary details

Rating 171 (v) <sup>629</sup>

(a) packet thousand pages ... (May 4) Even Serre is no exception to the rule, having



long (as André Weil) developed a tendency to declare that math who have not the good fortune of interest are "bullshit". He and Weil are nevertheless a format which (one would think) should put them above such childish. In this case (and apart from the "last twenty pages" of Deligne) is two or three thousand pages of "bullshit" grothendieckiennes the Weil conjectures eventually be proven (and a lot of other things too which Weil nor Serre never dreamed). This has not led to Serre more modesty, as in the text where he exhibited the demonstration by Deligne the last step in these conjectures (in the Bourbaki seminar in February 1974 exposed n ° 446), it takes this opportunity to irony among all (in polite terms, it is understood) on unnecessary details that must be drunk the "1583 Pages" SGA 4. In this easy irony, I do not detect a malicious nor bad faith, but unconsciousness and lightness. It will bother to raise the number of pages of three volumes (that he kept reading and the substance of which escapes him) and make an addition - history to care with "elegance".

coherent duality theory to a theory that encompasses the complex of differential operators (see note b. p. (\*\*) page 946). There is a reference to the Fourier transform p. 2 of the introduction to the statement "Poincaré duality" of Z.Mebkhout, in seminar on Singularity University Paris VII (1977-1979).

626 (\*) This is the "review" implicitly clear from the Introduction to the Symposium, already mentioned, and signed by Teissier Verdier.

627 (\*) For details about sleight-scams My friend Peter about the paternity of the theorem ever named, see note last year "The Conjurer" (n ° 75 ).

628 (\*\*) (May 19) For details about the misadventures of my friend Zoghman candidly lost in an environment of "hard" to nines and affable air, see the end of sub-notes "Hatching a vision - or the intruder", "The Mafia", "Roots" "Carte blanche to pillage" (n ° 171 (i) 171 (iv))

629 (\*\*\*) This paper (in three parts (a) (b) (c)) is the result of two notes of b. p. in note "ancestor" (n ° 171 (i)) - see Notes b. p. (\*\*) p. 944 and (\*) p. 945.

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But everything fits both my complacency once vis-à-vis such brilliant students, this p. 965 "elegance" -There Serre (at a time when the past four years the Burial was going well...) 630 (\*), and all what followed. Three years later hardly, one would find in the writings of my non-student Deligne with malice and impudence and more, even under their Serre or implied, with these "details useless "we pruned," confused state "and" matrix of non-sense "(where the same Deligne learned his craft and found his main inspiration), a blade digest his pen is intended charitably "to forget. "Thus, complacency in ease and impudence are we arrived in the mathematical world, in just ten years old, to a state of manners in which the simple decency of feeling seems to have disappeared. These are neither Weil Serre, much less Deligne, who created the new tools that were missing for " **The Conjecture** ", but one where they like to ironise - by deliberate ignorance or maliciously calculated, the impact is not very different. But I, who, with infinite care, wrote and rewrote, and actually write and rewrite tirelessly throughout the months and years, with a text that exposes the full extent it merit, language and some basic tools for a wide unifying vision, new and fruitful - I know me, and in full knowledge, there is not **a page** from 1583 neglected by Greenhouse, by my students and by the unanimous fashion, which has been weighed and re-weighed by the worker and that is his place and there to perform its function, no other written page to this day can not fill. These pages are the product of a fashion nor a vanity, delighting to put themselves above others. These are the fruits of my loves and labors long and obscure preparing a birth.

For this part of my work, as also to all my major contributions to mathematical who now entered the common heritage, **nobody** until now has managed to duplicate what I did (with blows of "bullshit" of "unnecessary detail" and "non-sense"), except by copying me (to insignificant variations close) 631 (\*\*). Some recopy (as such or in related contexts or new) saying it (it starts to become more rare...), the other playing new fathers, and taking disdainful condescension airs vis-à-vis the work they plunder shamelessly, and vis-à-vis the worker who taught them their

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job. This indecency then could thrive and spread because it p. 966 found a ready welcome consensus, and this first and foremost with those who (by their stature often exceptional) set the tone.

**(b) machines do anything ...** Yoga of six operations is an integral part of this vast unifying vision "developed in seminars SGA and SGA 4 5. I would even say that yoga is the theme Central oral seminar SGA 5 or rather, he is the "nerve" and soul. Also Illusie he took care to remove publishing-killing (destined to become its "care" volume "digressions

techniques" . . . ).

In the note "ancestor" (n ° 171 (i), p. 945) I write (without elaborating) that the vision of the six-power operations "gave eloquent proof of his power." For me, the real sign perhaps the most striking be that power is in control we have of the étale. To arrive at this control in 1963, the vision "six operations" that came to me from the consistent duality was my thread constant. I also believe the only person in the world qualified to decide on what was instrumental in the development of this tool.

It is understood here that in the process of discovery, said members "heuristics" are almost always decisive. If I speak of the "power" of a point of view or vision (something of a different

630 (\*) (27 May) for a reflection on chaining the evocation of Serre, see part (c) of this note.

631 (\*\*) (June 7) I recently read the fine book by Fulton "Intersection Theory" ("Ergebnisse", Springer Verlag, 1984), and finds it appropriate to make an exception for the theorem of Riemann-Roch-Grothendieck.

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order a theorem by itself), it can not be measured in purely technical terms. It about above all of its power "suggestive" as discreet and reliable guide in the journey of discovery, we blowing sensitive times "" good idea to introduce, "" good statement to identify and prove, "the" theory that remains to be developed. It is to have forgotten such a vision guide (after buried) that makes in the cohomology theory of algebraic varieties, the powerful momentum of the sixties has resulted soon the years after my departure, to a state of confusion and stagnation. Apart from the great "prestige issue" (Namely that the absolute values of the eigenvalues of Frobenius), all key issues were stubbornly evaded. . .

As another sign of the power of vision (or in this case, the formalism) of the six operations I see the formula fixed points Lefschetz-Verdier, both in the context of discrete factors that consistent. Here the role of the "six operations" formalism was both **heuristic** (in the sense that the formula is suggested by irresistible

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formalism) and **technical** (in the sense that formalism also gives the

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necessary and sufficient for the proof of the formula). It is true, given the burial, a small only portion of cohomological formalism that I had developed was used, until all at least the "rush" on the intersection cohomology and baptized beams "perverse" (where part of the formalism is exhumed without mention of the worker. . . ). But I know, for myself, with speculation Weil and the ubiquitous topos intuition, vision of the six operations was my main source inspiration in my thoughts cohomological throughout the years 1955- 1970 632 (\*). This means that the "Power" of this vision is obvious to me, or rather, a reality I have experienced almost daily for fifteen years of my life of mathematician. This experience has also reconfirmed still vividly the last few weeks, as soon as I made contact with "yards abandoned" crystalline coefficients and De Rham and the reasons 633 (\*\*).

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This experience any "subjective" I have the power of a vision-strength, also has a

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meaning "objective", difficult to dismiss out of hand. This sense appears when we want to remember that (Apart from a few exceptions) the main ideas and concepts concerning the cohomology of varieties algebraic "abstract" and diagrams (that everyone uses today as if they went back to Adam and Eve 634 (\*\*)) were identified by none other than me, during this same period 1955- 1970. (It I understand that I put aside my starting point here FCC, and Weil conjectures).

632 (\*) (15 May) It is understood that the vision itself gradually took shape during this period, from the first germs in my 1955 article "On some points of homological algebra" (the Tohoku Math. Journal). She is reached full maturity in 1963 with the abrupt start of the étale. This occurs (coincidentally) in the same day, at almost exactly where I introduce the "functor missing"  $R_1$  (direct image own support). But the Role of six operations, such as "vision-force" and as ubiquitous wireless driver, only became fully aware, I think, with the seminar SGA 5. In 1966, with the start of the crystalline cohomology, it was clear to me that the first purpose (beyond the limited program "running-in", which will be accomplished in the thesis Berthelot) was to arrive at a formalism of six operations (and bidualité) for "good" crystalline coefficients. It took a crumbling (declared deceased) so the coffin prepared for him, for (almost came years later, and inspired ideas of an unknown wave service and co-buried) these "good coefficients" finally end up being only **defined** ! We find a description for finitely diagrams  $Z$  include, in Volume 3 of Reflections (with the fifth and final part of Crops and Sowing).

633 (\*\*) (15 May) for the image of "abandoned sites" (or sites "sorry"), see Part 6 of the Funeral Ceremony (Rating 176 ©177, 178), including the last of the three notes mentioned. It was enough that margin writing Crops and Seeds, I spend a few hours here and there the problem of crystalline coefficients De Rham and the reasons for

see a compelling explanation for the former, and at least a block for the construction of the latter, in the crucial context of such schemes over  $Z$ . (compare with the comments of the note b. Previous p.)  
634 (\*) About this mentality of "user" (or "consumer") for finished products mathematics, who has forgotten (if he ... never knew) that it is a creation, and also about Adam and Eve and God, I refer the reader to notes last year, "a sense of injustice and helplessness..." (n ° 44 ") and "The unknown service and the theorem of good God "(n ° 48 © See also "Failed to education (2) - or create and conceit "(note ° 44©  
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This resulted in big time, mathematically speaking (and from what I could see up present) on a gloomy mediocrity, whose root cause is in no way be at the technical level. It©a signs of mediocrity, a powerful vision made to inspire and nurture great designs, was buried or left to derision, by those even who had been the custodians and the first beneficiaries. And another sign that neither a Deligne, nor Verdier, nor Berthelot or Illusie, they met were yet by all facilities that provide position and prestige of brilliant gifts and experience consumed, has been able to do the work that was required on the coefficients De Rham, in line yet their own research (and vision challenged...); nor recognize the innovative and fruitful work, when it is seen facing. And it is in this **same** spirit (because everything fits again...) That Once finally recognized the scope of tools from the new work, they were quick to seize without even understanding it, and bury alongside the ancestor, the unknown worker who had fashioned. . .

(c) **Things that look like nothing ...- or dryness** (May 27) 635 (\*\*) The way

I am speaking about Serre came there spontaneously, and stems from a perception of things, the about who had to train me in the last weeks or months. There have been

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Yet p. 969

writing these lines, uncertainty or perplexity residue, or reserve, vis-a-vis what I had to describe. I did hear, in short, that Serre would have missed on this occasion "elegance"! The fact is that, for nearly thirty years I know Greenhouse, himself represented for me the incarnation Nation precisely " **elegance** ". I must not be the only one, surely, to perceive it that way. It©about a elegance, both in his work and in his work as in his relationship to others, which is not pure form. It also involves a scrupulous honesty in work, honesty and an equal requirement vis-à-vis of others. More than once I noticed her acuity of judgment before any hint of "interference" in such col-bequeaths unscrupulous, trying to evade a troublesome difficulty (for failing to recognize that knew how to overcome it), or any errors of his own. . . This elegance thus involved, too, a **rigor** , both vis-à-vis itself and others.

Are all those things that for me are inseparable from the person of Serre, who have inter-come to this "resist residue" in me that I have just mentioned, before the spontaneous expression of another perception of things, taking the lead unexpectedly on the familiar perception. It does not matter to want me aside one of the two for the "benefit" of the other. The one and the other have to teach me some thing, different aspects of a complex reality, and which also is not static. To me the position relative to the other, to achieve a nuanced understanding of a person that connect me a past, and feelings of sympathy and respect.

This "rigor" of which I just mentioned did not extend, however, to all that had to do with relationships Serre to mathematics and mathematicians. I called sometimes a "unconsciousness" or "lightness" I could as well call a " **closure** ". It is in contrast to the attitude of "caution and modesty "I met with most seniors, as Serre himself welcomed me with kindness in my early days, and sometimes (and such was the case) with heat. I speak about it later (in Note "Freedom...", n ° 171 (vii)), where I find that this attitude was part "of the atmosphere of respect... that permeated the environment that welcomed me. "

The "closure" I have seen in Serre, on occasion, does not date from yesterday. I perceive the Early signs in the second half of the fifties. I think she has much limited depth and scope

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of his work from the sixties. I feel a connection between this aspect of "closure", vis-a-vis p. 970

635 (\*\*) This third part of the note "Unnecessary details" comes from a footnote page to the first part. See footnote in Note b. p. (\*) Page 965.

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of different approaches to mathematics of his, and deliberate about which developed in him little bit, to lock up his understanding of mathematics and mathematical things in a view (or

"Blinders", would I want to write) purely technical or technocratic, closing themselves to anything akin a **vision** ; something, therefore, which would exceed the statement (or group of statements) tangible, immediate, **provable** or (in a pinch) in the form of conjecture "pure and hard" to the contours fully tran-  
kets, "close" on the whole (except he has yet to prove it...). Looking back, it seems to me that it ended up  
push to the limit this aspect of his creative abilities, appearance exclusively "yang" and "super-  
yang "appearance" **macho** ". Given its exceptional influence over the mathematicians of his generation, and two  
three others that followed, it seems as Serre has contributed much to the rise of technocratic mind  
excessive I see rage in the seventies and eighties twenty, only today that is still  
tolerated, whereas any other approach to mathematics has become the object of general derision.  
In the words of CL Siegel, there nowadays extraordinary "Verflachung" <sup>636</sup> (\*)  
a "flattening", a "narrowing" of mathematical thought, a private dimension - the dimension  
visionary, that of dreams and mystery, the depths - with which it never had before (it me  
seems) lost contact. I feel like a **drying** , a **hardening** of thought, losing  
His living flexibility, quality nourishing - became pure **tool** ,

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stiff and cold, for flawless execution

p. 971

Tasks "snatching" of tasks by public auction <sup>637</sup> (\*) - when the sense of purpose and direction, and  
the meaning of these tasks themselves as parts of a vast all, are forgotten by all. There sclerosis  
deep, hidden by a feverish hypertrophy.

This imbalance of thought is a sign among others of a more critical imbalance, and a vacuum,  
a deeper deficiency. It is no coincidence that this drying of thought has spread and installed,  
over the past two decades together have eroded customary forms of  
delicacy and respect in the relationship between people. And it©not a coincidence that wind  
contempt who stood up and I have finally felt the breath, was accompanied by a more or less corruption  
general, I do not finished for over a year to go around.

Serre until today still felt nothing of the corruption there, which surrounds on all sides. I him  
had known the fine nose, though. But all is not to have the fine nose, it is still necessary to use it, take  
knowledge of what he has to say, even when smells he speaks we are able to incommo-  
der; or, to worry, when they put ourselves into question. I know Serre, nor

<sup>636</sup> (\*) I take this expression (in German) a letter from Serre, received very recently. The term is derived from the preface  
LC Siegel to works Hecke. Greenhouse cites this impression of CL. Siegel at the very end of his letter, immediately adds:  
"It was unfair, and it would be even more now, I think." At home though the penny dropped and it continued to work.  
My short reflection on the relationship between Serre and I was probably out of there.

I also believe that if Serre cited Siegel, is that somehow this impression from a major ma-  
thematicians of our time, had to work in him already; it was like a quack, probably, in "life in pink" mathematical.  
A quack surely among others, but less easy to remove, apparently. . .

"Flach" in German means "flat", "lacking depth"; "Verflachung" refers to the process leading to such a state of  
"Flatness" or the result of such a process that has just taken place. In the main text, I endeavored to follow  
associations aroused in me by this term very telling, as is untranslatable, unfortunately. Of course, I know if fully  
how I perceive the thing is covered so slightly with the perception of Siegel, I have not read the text that quotes Serre.

<sup>637</sup> (\*) This image of "public auction" I must be suggested by ads for "tenders" (sic) which are riddled the  
"Newsletters CNRS" and other papers I receive periodically, as an attaché of fresh out research  
in this esteemed institution. This jargon, among many other signs, show how this "flattening" of the work  
discovery in no way limited to the middle I had known, nor mathematical science. I have not found yet  
for tenders in pure mathematics call, but it will be soon - and I can imagine myself as my old friends and students,  
seriously sitting behind padded doors, in such daunting committee acronym, to decide which "research areas"  
priority must be reported, what "outreach strategies" to promote, and what "offers" teams "classified Winning" he  
should "hold" to a "screening" or even, honoring the jackpot, the official grant by the Ministry of  
Guardianship, renewable every two years after favorable opinion of the Competent Commission. . .

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## 18.5. THE FOUR OPERATIONS (a corpse)

me, would dare to howl with the wolves, loot, of scheming and run down there or "everyone" plunders,  
skulduggery and débine. There is none of this, of course - he just hold your nose (and so what if suddenly  
it has a hand under. . . ), And to one who felt nothing.

And it is in good company - not one of those who were my friends, in this world that we might  
common and whose smell reaches me into my retirement - not one spoke again, it was by  
referring, odor

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he would have felt and that would have bothered. Many still surely remain those <sup>p. 972</sup>  
among my colleagues who continue to exercise faithfully mathematician profession, which deserves this

respect. But among those who sit in the first places, I know of **no one** who had that simplicity to believe the testimony of his healthy faculties (olfactory, in this case), rather than their noses not to have to say something smells bad here - it would maybe go see. . .

But I return to the person Serre and mine, and this "closure" I felt at home, appeared I do not know when and who went to accentuating the years. I think the most fruitful part of his work, one that has most profoundly influenced the mathematics of his time, up to the beginning, before the appearance of the closure or at least before they have taken a decisive grip on his relationship mathematics and mathematicians. It was in those years, too, in the fifties, the contact with him was for me the most fruitful, it is in these years that is up this role of "detonator" that Serre played with me, giving my work some of his most decisive impulses. It in those years also born and has grown in me a broad vision that inspired my work and fertilized in these years and until today. I can say with full knowledge of the facts, that there has been someone besides me who had a hand in the outbreak of the vision, it is he, Greenhouse, and in those years. And he could be and because in these fruitful and decisive years, there was an opening in him to mathematics things for what they are, including those still left the immediate decision; those who seem reluctant at first to leave by identifying the cracks already formed language - those ask perhaps obscure years and labors patients, if not a lifetime, before condensed into tangible substance and reveal the members and the shapes and contours of a **body**, alive and strong, attesting the appearance unexpectedly in the familiar context of the known, a **new being**. I think in the first years when I met Serre and until the late fifties, he kept sensitivity to this impalpable thing and delicate what "creation", and for the humble labors that prepare a birth. I think at one point he was able to feel the birth of a vision, and language that it gave shape like the soul or spirit, and body. . . There was then a heat without speech, availability discreet and efficient, where he could assist an arduous and intense work that was not his, and which Yet, for a sympathy and an expectation, he participated.

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I can not say when and how this liveliness in him, the level of our common passion is p. 973 blunt, has given way to something else, I tried sometimes to identify. Already at the beginning of the sixties if not before, he stopped receiving the forest, to consent to see that this tree or that he found to his taste. The rest had no place. It just annoyed, I think, to see me so absorbed in tirelessly clearing large areas without color and plant there patiently all these things that still looked like nothing, with the air of one who already see a thriving forest 638 (\*).

That does not prevent me to continue clearing, planting and replanting, pruning, and redéfricher and 638 (\*) (June 17) Of les six "yards" that I review in the note "The tour of building sites - or tools and vision" (n ° 178) there had u **n only** (the site "grounds") who has had the good fortune once to interest Serre so slightly - and again ... When I wrote recently without comment, in a PS, I thought I had the principle of shaped construction the category of patterns on a scheme of finite type over  $Z$ , no alludes in his response. Certainly these "math grothendieckiennes" do him neither hot nor cold...

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replant - nor is it always as friends and we spent hours and hours discussing even Math (phone, usually). When I had a clear-cut issue, and an issue was not in the index, it was for him especially since I used to speak, for once he lights - and often, in fact, he had. I continued to learn a lot from him, and surely he learned by me then that could be of interest. It was better that an exchange of good processes or services - there was always a passion that connected us, there was fire and spark. But he had already ceased to be for me an inspiration. This source was now in myself only 639 (\*\*).

## a2. Questions absurd

**Rating** 171 (vi) (May 5) 640 (\*\*\*) My memory here was a blur, and has redefined the weeks following, where I had the opportunity to reconnect with so little issues. There were actually two separate questions in my mind, a perfectly precise, the other quite vague. The first question concerned the need to reach a complete theory of the six variances for "Coefficients Rham" that remained to define precisely. My crystal ideas, both in feature  $p > 0$  in characteristic zero, provided a very precise beginning - we already knew, in advance, what which was to replace the "local systems" (or "bundles

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twisted constant ") l-adic (or Betti in

p. 974

transcendent frame), and they had come to define "coefficients with singularities" in the spirit of categories Derived course 641 (\*). What was missing so it was a good condition "finitude" for complex crystalline. In characteristic zero, it is the "3-coherence" (to which neither I nor any of my students has thought,

while it is an idea so simple and natural!), combined with the delicate Holonomy terms and regularity, which gives the answer, as we learned (twelve years after the start of the crystalline yoga) the Philosophy of God Mebkhout alias. I look as if curiosity of my former students will eventually move (Without naming the unknown service or ancestor is heard a thing ...) to identify the conditions 639 (\*\*) (June 12) for a continuation of this reflection about the relationship between Serre and I see the note "Family Album" (n ° 173), part c. ("The one among all - or acquiescence") of 11 June, and parts. summer. 640 (\*\*\*) This note is from a note b. p. in note "ancestor" (n ° 171 (i)) - see note (\*) on page 946. 641 (\*) It is also clear, when the base body was C, we wanted a category equivalent to that complex of faisceaux C-vector to algebraically constructible cohomology beams. This indication of a high precision suggested that by unscrewing, the critical issue was that of associating at any crystalline local system on a sub-diagram (not necessarily closed), a crystalline beam on the ambient diagram. This is essentially what was done by Deligne in 1969 it except it turned out that instead of a crystalline beam it was a **pro-beam** lens, which then was an idea important new (and "obvious" as soon as one takes the trouble to look ...). But the systematic work with pro-objects would have required a work considerable foundations, including the one made by Jouanolou for his thesis (on the coefficients l-adic) gave a taste. It should have been there up his sleeves again. . .

The new approach Mebkhout by 3-Modules back since (from the perspective of Deligne and mine), replace a pro-beam lens with a crystalline ind-beam (through the functor dualizing coherent **ordinary**  $\text{Rhom}_{O_X}$  (-  $O_X$ ), and **pass the inductive limit** for a regular crystalline beam, ie (assuming now X smooth on a body of characteristic zero) March 1  $X$ -modules. The "miracle" so unexpected, established by Mebkhout between 1972 and 1976 (starting from a "bout" opposite, cf. the note "The three milestones" n ° 171 (x)) is that the 3-module is **coherent** (specifically, beam coherent cohomology). Another miracle equally unexpected is that one can characterize the 3-modules (or rather, the complex 3-Modules) that we obtain by simple conditions, completely new nature with respect to the optical crystalline grothendieckienne (namely the condition "microlocal" Holonomy, in addition to a condition of "regularity" introduced by Mebkhout and become familiar meantime).

(May 26) For details about the duality relation between coefficients De Rham - Mebkhout and coefficients From Rham - Deligne, see note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), part (c). For the need to replace the perspective of Deligne procohérents modules by the crystals into coherent promodules, and the possibility (not proven yet) replace the bulky viewpoint of pro-objects (crystalline or laminated) beams by crystal no more (by passing to the projective limit), see the same note, parts (c) and (d).

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corresponding bus.  $p > 0$ , or rather probably, in the rigid-analytical context charac

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teristic p. 975

nothing. Better late than never. . . 642 (\*).

I did not pursue this issue myself in the sixties, with enough other spots and thinking that with Berthelot and Deligne in the running, she was in good hands (like what we can wrong. . . ). The work of Deligne in 1969/70 yet provided a response in principle feature no, that would probably satisfy me if Deligne had conducted this work forward. But in my mind, such a conjectural theory coefficients De Rham, even if it were to Related cohomology "discreet" (in the form of a crystalline cohomology) and cohomology "coherent" not "wore" not provided consistent duality theory. So I did not see a coherent beam zariskien defined a "crystal enveloping" 643 (\*\*) (NB in the language of 3-modules, it is the extension of the scalar of Ring  $O_X \rightarrow 3_X$ , X for at least smooth. . . ) - and although I had seen, the resulting crystal (already for  $F = O_X$ , which gives the crystal  $3_X$ ) are **not** the type of Rham. Still, I wondered if on a complex analytic space X, the coherent duality (e.g. in the form of Serre, if X is smooth and for locally free coefficients) could not be obtained as a "special case" of duality discreet, developed by Verdier on the theory model spreads. As such, it looked a little goofy and immediately raised a host of questions: how to explain "in discrete terms" the role of the module Dualising (differential forms of highest degree)  $\omega_X$  and how to account for évétesques pathologies which had no analogue in duality "discreet"?

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This is Mebkhout who was the first (and only until today except me, he seems to) understand p. 976 that there is indeed a deep connection between the two dualities, but that it is only expressed **not** saying as a "cap" the other, but by finding a third duality of theory 644 (\*) one of the  $3_X$ -modules (or "Crystals" X), that "cap" one or the other, and by limiting, in addition, the "discrete" side, of the complex C-vector that are cohomology sheaves **analytically constructible**. There is no doubt me that this is "the correct answer" to this "vague question" (and a little off the plate...) I had

Yet never had the opportunity to ask my posthumous student. . .

(15 May) The writing of "The Apotheosis" became at the same time an unexpected opportunity for me familiarise so slightly with the work of Mebkhout and yoga 3-Modules that he introduced in the study cohomological varieties. Along the way, it was also back memories that had sunk. I

I have noticed that especially in the late fifties or early sixties, I had was closer to the "philosophy Mebkhout" than I was aware to me there are only ten days, by writing the beginning of this note ( "Questions absurd"). Under the own patterns and Smooth on an arbitrary basis, I had in my hands a duality statement (in terms of a complex operator differentials relative and complex "Assistant"), "capping" the consistent duality and duality for cohomologie De Rham. Technically, it was pretty much the equivalent of algebraic version

642 (\*) (26 May) Since these lines were written, and as unexpected fruit of my efforts to make an account of which the Apotheosis be worthy to be remembered, I was led to clear (without almost on purpose) which I think is now **the** good definition of the coefficients De Rham, at least for a scheme of finite type over Spec Z, (which appears to me as the The most crucial event of all). Of course, the essential new ingredient, compared to my 1966 ideas, is the philosophy of Unknown wave, I refrain (like everyone else) to name here.

The approach that I plan to type schemes finite over Spec (Z) must also give the right coefficients From Rham (Mebkhout or Deligne style choice) for type schemes finished on any body (characteristic zero, or no). I plan outline this approach in the "Coefficients De Rham" Volume 3 of Reflections, among others "Technical digressions" that my students can come and copy at home. . .

643 (\*\*) (May 26) It may be worth taking the "co-crystal" wrap (see note 171 (ix) Part B, for allusions to the concept co-crystal). I will no doubt on this issue in the explanatory promised in the previous footnote b. p.

644 (\*) For details about this "third duality theory... Which covers the other two," see footnote. "This work..." (n ° 171 (ii)).

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Mebkhout duality theorem (which was discussed in the complex analytic context in note "The work of...", N ° 171 (ii)). Yet my duality statement did not satisfy me, and I have not thought about the publish or even to make it to advertising because it seemed to me, in the form called, too close to the theorem duality Serre (in perspective on an arbitrary basis, it is a thing heard), which is a corollary more or less immediate. To arrive at a statement that satisfies me, it would have as I know how to do a "derived class" with complex differential operators, so as to make a statement intrinsic duality in terms of objects of these categories, on the model of coherent duality theory released in previous years.

What was missing, so it was a good concept of "quasi-isomorphism" to a morphism (differential) between complex differential operators,

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so as to form a derived class (reversing formally

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ment these quasi-isomorphisms). It was clear that the usual definition (via the beams cohomology associates) was not used in the algebraic framework (and it is not probably the most under transcending 645 (\*)). The passage of the complex corresponding 3-Modules now gives an answer wonderfully simple in my perplexity of yesteryear!

Seeing no ready definition for the concept of quasi-isomorphism, I have not tried then to get to the bottom if it existed or not, and whether there would be indeed a remarkable derived class. It was at a time when I was the only one interested in derived classes (though much less sophisticated) formées from coherent modules and morphisms **linear** therebetween. . . I did not feel clearly that the issue of a concept to emerge from quasi-isomorphism (also a bit vague, not to say farfetched) drew to a fruitful mystery, which mystery admitted a "key" childishly simple! and he there was a category of "coefficients" remarkable waiting only whether defined. Should have been for this, no doubt, my thoughts continue in an atmosphere where they meet a minimum interest and echo, were it only in a party that is involved!

This is the De Rham cohomology that attracted my attention on this fact, evident of course, that the spaces global cohomology of coherent sheaves on an algebraic variety X over a field k say, are of "functor" not only with respect to homomorphisms  $O_X$ -linéaires, but even with respect to **all** the homomorphisms of k-vector bundles, particularly for differential operators. It is this observation that motivated an embryo of reflection on a "coherent" duality theory (or "quasi consistent"), where" morphisms "Beam would be differential operators, rather than linear.

This reflection was cut short, as I said, and this even to the point that it did not remain in me corner of memory, as a thing (among many others) should be

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one day clarify

p. 978

- it sank (I think) in a total oblivion until the last few days. Even my reflection sporadic on crystals, around 1966, did not go back in my memory, as far as I remember. Yet, this crystalline reflection, that I do not doubt me then (while failing to remember only the question!), would provide me since 1966 a **different** key, "dual" in some way to that of Mebkhout for my old perplexities through the complex of the main parts of infinite order associated with an complex differential operators. I am referring in a note b. p. written yesterday (note (\*\*\*) on page 946) and I intend to return in detail in the section of Volume 3 of Reflections, developing yoga "types coefficients" and giving such a shape definition of what I presume to be "the" 645 (\*) I am wrong here. Mebkhout guarantees me for a homomorphism (differential) between complex differential operators, it is a quasi-isomorphism (naive meaning complexes associated C-vector beams) if and only if the homomorphism corresponding to the associated modules 3-complex is a quasi-isomorphism. This is equivalent effect (At passage mapping-cylinder) to say that a complex differential operators is almost zero in the naive sense, and if only if the associated modules 3-complex is almost zero, something apparently well known (at least to Mebkhout who demonstrates in his thesis inexhaustible. . . ).

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De Rham good coefficients (Mebkhout style, or Deligne choice) on a scheme of finite type over  $Z$  (for example).

Technically, even "psychologically" (in terms of the problems already then, and vision overall that gave them strength and life) everything was ready in the second half of the sixties, to clear the definition of the coefficients De Rham. Deligne after I was on the verge of good concept, and he could not help but disengage if a force to which he gave absolute power over his life and on his work, had been premature and conclusive end to his thoughts in this direction. . . 646 (\*) Discover, it is not typing on a nail or chisel, or on a steel wedge, tooth and nail and strokes hammer or mass. Discover is above all, to listen with respect and with rapt attention, Voice of things. The new thing does not spring ready-made diamond, like a jet of sparkling light, nor does it spell a machine tool so advanced and powerful it is. It does not look noisily, studded with acclaim; I am this and I am that ... It is a humble thing and fragile, delicate and living thing, a humble acorn which may release a oak (if the seasons are his propitious. . . ), Or a seed that will create a stem and the latter to a flower. It is not born under the limelight, or even the sunlight. It is not the fruit of the known. His mother is the Night and dark, elusive mist and without contours - the prospective escapes the words that would identify the absurd issue that is still looking, or

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Such dissatisfaction so vague and elusive and real p. 979

Yet, with that indefinable feeling (and irrefutable...) that something is wrong or is askew and there is something fishy. . .

When we know humbly listen to those voices that speak in low voices, and follow obstinately passionately their elusive message then - after groping and obscure labors, muddy perhaps without appearance - suddenly the mists incarnate and condense, in **substance**, firm, tangible, and **form**, visible and clear. In this solitary moment of intense attention and silence, the new thing, the daughter of the night and mist, appears. . .

#### a3. Freedoms. . .

**Rating** 171 (vii) (May 4) 647 (\*) I do not intend to ask the man "mature" or "wise", surrounded by immaturity and irresponsibility of his kind - it is not, I imagine, the picture that emerges from my person the pages of crops and sowing 648 (\*\*). Yet in my relationship to mathematics at least, I think to say everything in my life was maintained simplicity of good quality 649 (\*\*\*) at the same time 646 (\*) See in this regard the reflection in the sub-note "... And hinders" (n ° 171 (viii)).

647 (\*) This sub-rating comes from a note b. p. in note "ancestor" (n ° 171 (i)) - see note (\*\*) on page 945.

648 (\*\*) (May 26) I can even say that if writing Crops and Seeds has revealed something about it, it is a state to "immaturity" in fact, a lack of "wisdom", and not the opposite. This was perhaps the most unexpected discovery of all, and most crucial also for its immediate implications, the strength of my attachment to a past and my Artwork mathematician. This attachment, as relatively discreet, was first revealed to me the end of March the year Last, in the reflection in the final note "The weight of the past" (n ° 50) Fatuité and Renewal. It is to be faced with the brutal reality of the Burial in aspects especially willful disregard and violence, which has set in motion me strong egotistic reflexes of defense. They reveal to me at the same time the power of the ties that bind me to a past, which I have thought once he was alienated from me. During the past year, these links appear to have taken a new force, and often (especially lately) I feel like a **weight** indeed a grueling weight to true say - like other weight that weighed on me once, and that resolved. . .

649 (\*\*\*) (May 16) I would have to do here except a certain possessive attitude vis-a-vis my "turf" on which I put the finger in Fatuité and Renewal in the "Sport mathematical" section (n ° 40). This provision



"Sports" should take me to minimize the ideas of others, whenever they were already known from my side. We can say (contrary to what I said in the main text) and in these cases, my vanity indeed interfered with "My sound judgment", and tended in such a case to make me a discouraging attitude, where a benevolent encouragement

a loyalty to my nature

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original. Vanity, who was also pervasive in my life and in that of

p. 980

any of my colleagues do not yet interfered (as far as I remember) with my healthy judgment and my math acumen <sup>650</sup> (\*).

It was also after I left in 1970 I started to realize gradually and each

Once in amazement, at which point it is common even among men of exceptional ability,

that they are sometimes as annihilated, hopeless blocked, it would seem, by prejudices

of "irrational" in nature - and even more stubborn! My first experience in this direction are placed

1976 <sup>651</sup> (\*\*), and are discussed in note "You can not stop progress" (n ° 50) and a first written reflection

about continued in the note "The Gravedigger - or the whole Congregation" (n ° 97) <sup>652</sup> (\*\*\*), in

the particular context of

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the Burial. It is only gradually, and against inertial forces

p. 981

huge, I realized that these causes "irrational" are not less perfectly

intelligible, if only one bothers to stop and to probe it. It is through this that I ended

by "accept" too, somehow. . .

Going back to my person and my relationship to mathematics. Through my work style, I tend

to operate strokes often hasty presumptions, without worrying about "prudence" <sup>653</sup> (\*); but I am

through each of intuition (or "presumptions") appeared, making that many mistakes

dotted throughout the early stages of labor eventually eliminate, to make way for a

understanding a rock solid, and (usually) key indeed in the heart of things.

My spontaneous way is any other when it comes to pass judgment on other people@work,

and especially when it is placed in a topic or records with which I am not familiar. I have

tended then, I think, to exercise caution and modesty. It was also well

here the example that was given to me by most seniors who welcomed me among them, such Cartan

Dieudonne, Chevalley, Schwartz, Leray - to name a few. I do not remember hearing

none of them speak authoritatively, whether bad or good, on a work whose substance

escaped them. This caution, I now realize, was part of the atmosphere of **respect** that I spoke

Moreover, that permeated the environment that had welcomed me <sup>654</sup> (\*\*). It seems to me that it is this caution sign a

respect, which deteriorated in first in this environment which I identified myself for over twenty years

my life. Perhaps my memory betrays me and I am deluding myself, but I think I was relatively

would have been appropriate. It seems to me that such situations were exceptional in my mathematician@life, and that

did not represent an obstacle in my mathematical creativity.

<sup>650</sup> (\*) See Note b. p. Previous to reservations about it.

<sup>651</sup> (\*\*) (May 16) It@not really my first experience in this sense - I had done more in the years

previous with Deligne particular, and also in my past before I left. But those experiences remained

sporadic, while the episode around the thesis Ladegaillierie was impressive by the perfect harmony in

acts and omissions of five mathematicians (all high-level), which surely had not coordinated among themselves. It@here

My first contact with the Burial, beyond the vicissitudes of my relationship with the only person of my friend Peter.

But this extraordinary weight factor "irrational" in "scientific" called thinking goes far beyond the context

the Burial, and even of an era. It is not necessary to be versed in the history of science (and I do

am not) to realize that it is marked at every step by the effects of a huge inertia, opposing

to the outbreak of any innovative idea and fulfillment, however when the idea arose. For reflections in this

meaning, see especially the first two parts of Fatuité and Renewal ( "Work and discovery" and "The Dream and the Dreamer"),

Sections 1 to 8.

<sup>652</sup> (\*\*\*) This reflection is greatly deepens in "The key to the yin and yang", including in both ratings (concerned

ing the same "congregation") "The providential circumstance - or Apotheose" and "disavowal (1) - or recall" (n ° s 151,

152). See also Note "Muscle and tripe (yin yang bury (1))" (n ° 106) which opens the lengthy reflection on

yin and yang.

<sup>653</sup> (\*) About This style of work, see especially note "brothers and husbands - or double signature" (n ° 134), and also the

section (in Fatuité and Renewal) "Error and Discovery" (n ° 2).

<sup>654</sup> (\*\*) See "Welcome Abroad" section, n ° 9.

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## 18.5. THE FOUR OPERATIONS (a corpse)

unaffected by this aspect of the degradation of an atmosphere of respect. I always remained conscious, I think the extent of my ignorance in mathematics in general, and my limitations for power apprehend such work of others, when it was located outside my home interest, strongly centered most of the time.

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As for the work of others that I was able to understand and thereby to appreciate or judge p. 982 (Provided only that I want to give myself the trouble), I do not remember either error rough judgment, whether bad or good, I would have had to find afterwards. It is still The same feeling I had about my own ideas and intuitions that sentiment regarding the presence (or absence) of a "good question", or that of a rich substance to sound, or the scope of such idea, or understanding more or less complete and more or less profound that I had a situation or one thing. In all these cases, if error there was, it was each time in the direction instead of a "minus". Yes - usually the richness of a new or novel idea theme, its true depth range and in extension, only revealed fully only gradually, over weeks and months, when these are of years. This gradual confirmation of just initial feeling (usually), but remains first vague and diffuse, with a "work room" more or less deep and more or less thorough, we just then as a surprise and as a wonder, constantly renewing itself over hours and days. This, surely, the cause of the extraordinary fascination of the research (whether mathematical, or other): at each step, the reality that is revealed to us beyond our own dreams more reckless, in wealth, delicacy and depth. . .

But I return to my understanding of the work of others, when it was located in topics were familiar to me, even "hot" topics for me. I think I can say that my alertness to anticipate the true scope of an idea (which often escapes the author himself) played a role in my work capital. I am thinking first of all here in exceptional role Greenhouse, and the fact that during these Fifteen years of exceptional richness in my work, between 1955 and 1970, most of my ideas, and Most also my great investments, had their starting point in some idea or approach Greenhouse, sometimes seemingly innocuous. I intend to talk about more detailed way in the "Comments historical "in the Thematic Sketch 655 (\*). But it is not there, however, an opening particu- die vis-à-vis the only person Serre. The same (relatively speaking) occurred with other mathematicians, both in my past systems analyst, as in that of surveyor 656 (\*\*).

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I can say that, throughout my life mathematician, I was amply "rewarded" for p. 983 this simplicity of approach to mathematics, I just try to understand a little. This simplicity, 655 (\*) These "Comments" are advertised in "Compass and baggage" (Int. 3).

656 (\*\*) As an example (among many others), I point the principle of reduction of statements schematic situations on "finite presentation" on any basis, in case it is the specter of a local ring **finished** (or even a finite field), principle-reaching that I extracted a striking demonstration idea of a remarkable result (and very particular) D. Lazarus. On this subject the note "Go!" (n ° 77) and Note b. p. (\*\*\*) p. 297 to it.

(May 16) I am not sure when I inspired me with an idea due to others, I have been careful to report. For example, I do not remember, in the relevant paragraph of EGA IV, have taken care to mention Lazarus, as the source of the method general reduction that is developed there. This was negligence, which in those days, did not seem to get a result. I think people like Dieudonné (co-editor of the EGA with me) or Serre, who had to know like me this result Lazarus, as (probably) the first of its kind, would not have not considered compelling (or only appropriate) to quote - it was at least not in the canons of Bourbaki style! It is true that Bourbaki made amends in historical notes, which are lacking in the EGA and elsewhere in my work. Today, instructed by the frightening deterioration scientific ethics mathematical environment in years 70 and 80, I would be much more meticulous than I did was to carefully indicate my sources, not only in the technical sense, but also heuristic sense, which is often more crucial. In the "Comments" Historical already mentioned, I think at least repair some of my omissions in this regard.

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which in other areas of my life often told me: is lacking, is a blessing in itself. Actually, the fertility and power of my work due to this simplicity there, which is none other than as e nfant ...

## a4. . . . and impedes

Rating 171 (viii) (May 4) 657 (\*) I am wrong here, and my memories are specified (and ground) during the two previous months, taking up a little better contact with the subject. In fact, the main purpose of Deligne was precisely to give this "purely algebraic description" discrete beams (C-vector) Building and category suitable derivative 658 (\*\*). The coefficients that introduced (via a condition "Constructability" ad hoc on a pro-crystalline beam condition defined by the existence of a

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"Unscrewed"

p. 984

appropriate, based on the one that I introduced in the context spreads or complex analytic) are facts "measure "to meet that wishes. Therefore, it became (heuristically)" obvious "that a formalism six operations **must** exist for these coefficients (in characteristic zero), and it was even able to de-show rigor, "brutally and stupidly" by judicious application of the "principle of Weyl" reduction if (known) where the base body is C.

Therefore, it may seem a mystery, little one stops there, a Deligne could abandon obviously promising approach to the description of "categories of factors" that (this was a clear thing in the mid-sixties) would have a crucial role to play in cohomology of algebraic varieties. So he left to another the task of finally clear, eight years later, an approach somehow dual and penetrating <sup>659</sup> (\*), which was once <sup>660</sup> (\*\*) renew the theme Cohomological geometry. The thing was not so much struck previously, as this primer theory by Deligne was placed shortly before my departure, and that nothing yet, at that time, could have let presage what would happen to him. After my second start, and practically until very recently months ago, I had completely lost touch with the cohomological theme.

I thought lately, a bit hastily and without stopping, that the reason for this disaffection Deligne for a theory in which he had invested for a year, could hold the fact that was not satisfied with his standard-definition "constructability" by unscrewing. This might seem too simplistic, and it is a fact that is surely shallower than the local algebraic provided Holonomy and regularity, released by Mebkhout in 1976 in his view "dual". But on reflection, this "Explanation" simply does not stand up! This is certainly not because one approach to a critical issue would be "too simple"

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a mathematician in full possession of his larguerait

p. 985

and approach, and the question! At most he larguerait its first approach, the day he would have found another that allows to achieve a deeper and more complete this very issue <sup>661</sup> (\*)!

<sup>657</sup> (\*) This note is from a note b. p. in note "ancestor" (n ° 171 (i)), see note (\*) on page 947.

<sup>658</sup> (\*\*) It is here, category (denoted  $Cons^*(X, C)$  in the note "The work ...", n ° 171 (ii)) formed complexes faisceaux C-vector  $x$ , to analytically constructible cohomology sheaves, seen as full subcategory of  $D^*(X, C_x)$ .

<sup>659</sup> (\*) I have no doubt that if indeed Deligne had not dropped the theme of the coefficients De Rham (he held from me), he could not help, in the trodden, to discover (eight years before the unknown service) yoga "dual" 3-Modules, and familiarize themselves suddenly with the ideas of Sato School.

<sup>660</sup> (\*\*) The term "immediately" is not quite the reality as it was (but that, rather, "should have been" if...).

In fact, three years elapsed between the time the new philosophy and the new tool were ready, and when people that set the tone eventually realize that there was something that could be used (and pocketing good...).

<sup>661</sup> (\*) In fact, in this case, it seems to me that there is no place to "drop" approach Deligne in favor of that of the God (not to name Mebkhout). The two approaches complement each other, the one having the advantage of Deligne  
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### 18.5. THE FOUR OPERATIONS (a corpse)

When I ask so little about this strange situation, it becomes clear that in this case again, as in many others, the motives of my friend Peter had nothing in mathematical or even "rational".

In resongeant there, I realized how the problems around the coefficients De Rham, which made sense only from the perspective of six operations and lens Yoga <sup>662</sup> (\*\*) (yoga that I had introduced a few years before with crystalline topos, and in the spirit of just six operations. . . ) - at how this whole issue was rooted in my work and in my person, and this so

**clearly apparent to all .**

It is true that the problem of coefficients Hodge also had the same master, which already in his heart (and his own blind, perhaps) the student is distanced. But the descent was there less obvious to the outside world (and no, even including Serre, seems to have received <sup>663</sup> (\*\*\*)) and especially: a first tranche of work reaching which was to do, was not part of a vision ("Six operations" or whatever...) Ostentativement grothendieckienne, not so clearly apparent to all, at least.

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But this is no coincidence, as I have emphasized more than once that the cohomological theory Hodge- p. 986 Deligne, after a spectacular start in the late sixties, still remains at the stage childhood, or only tolerated coefficients are constant (or, at best, "smooth", ie the equivalent under "Hodge-Deligne" local systems), and where such crucial operations that direct images

senior Leray R i f \* (to mention only a few) do not run! The question of defining the right concept of "Hodge coefficients" and the relevant transactions above, is not only **mentioned** in Deligne's work (as far as I know), when she was already familiar to me, I believe, soon before I have had the pleasure to meet him. When, after my departure, and over the years, I sometimes ask (I ended up tired, of course...), of what he expected to develop at the end of the purposes theory that was needed, he invariably replied: "it's too hard..." 664 (\*). That does not convince me, that for sure - if I had gone in a completely different adventure, I should have been dry well also to expand this theory "too difficult" and that the coefficients De Rham the same time. . . to be closer to geometric intuition, and the Mebkhout being technically simpler (avoiding the use of pro-objects), and various deepest respects.

662 (\*\*) I remember also that in the statement that Deligne was his theory, he avoided the systematic use of language crystalline, which nevertheless gave her a deeper dimension theory, by inserting it into a cohomological formalism topologique existing. Also, I realize that, as Berthelot and cohomologists my other students, he had lost sense of **oneness** deep between the crystalline cohomology in characteristic p, and crystalline phenomena characteristic zero (which were the subject of the conference). These are signs of a deliberate ignorance of a land unit, which is morcellée arbitrarily and thereby destroy. This is deliberate in nature of a "blocking", by intervention natural forces ego, foreign to the drive for knowledge. For an illustration of this blockade in another of my cohomologists students, I experienced yet endowed with a fine feeling, see subnote n ° 91 2 to the note "The heirs..."

663 (\*\*) This seems out anyway Serre report on the work of Deligne, quoted in the sub-grade n ° 165 1 to Note "Requiem for wave skeleton" (in particular p. 813). For an explanation of this relationship, see "dot the i" (note n ° 164), I 4 (in particular p. 793), and its subnote n ° 164 1 .

664 (\*) This reply has recently partnered with "In Praise of Death" (or burial by the compliment), the pen Deligne, which was discussed again recently (see note "Jewels" n ° 170 (iii)). This "Eulogy" ends with this question (worth its weight of Peter...)

He left IHES in 1970 at a time when his passion for mathematics was eclipsed. Are we to believe that the problems that arose in the line he had drawn, **had become too difficult** ? "(My emphasis))

This kind suggestion is included in the part 2 of the praise, stone dedicated Deligne, where we learn that some conjectures of the deceased, "today still unaffordable then" had probably been (at least this is clearly suggested) the main obstacle has been overcome Deligne said, to prove some conjecture "proverbial difficulty." These comparisons make me understand that the stereotype answer "it's too hard..." My friend Peter, there was a subtext of derision that would give him satisfaction all the more piquant, it was evident that this goof of deceased was a thousand miles from suspecting said implied (not more than the quality of the deceased...).

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Looking back, I am struck by the parallels between the stagnation in the Hodge-Deligne theory of hand, and the other aberrant behavior Deligne vis-à-vis the theme of the coefficients De Rham (attitude highlight

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in the "perverse" inequity that remain attached to the memorable symposium Luminy June 1981. . .). p. 987

These two outliers appear to me now closely linked, and this to a whole other level though the mathematical level. It is true that, obviously, the development of a formalism coefficients Hodge is **subordinate** to that for coefficients De Rham (something that was obvious to me from the 1966, and people seem to be discovering for a year or two, on the broken student work-posthumous-ever-appointed ...). This mathematical fact makes more striking, both link between the two sets of facts, and the aberrant nature of the one and the other: because the link "objective" was a power additional incentive (for someone at least "in full possession of his faculties") for develop and the one and the other theory, which could, therefore, that illuminate and reinforce each other. Stagnation in one and the other theory (until Pervert Symposium 1981 Rham and up today for Hodge) is largely in the general slump cohomological theme, slump which I had occasion to allude more than once 665 (\*). Even without the spiritual dimension of the human being, and considering only the only factor "profitability" by scientific production "peak", this stagnation illustrates for me a strikingly both the empire unexpected that can take the occult forces on an egotistical being, and this even in the exercise of so-called science "disinterested" and the character (apparently) absurd of this empire that here (at first to least) seems to constantly go against the aim pursued 666 (\*\*).

### **b1. The five photos (crystals and 3-Modules )**

**Rating** 171 (ix)

667

665 (\*) On this mess, see especially "The yards sorry" (The Funeral Ceremony, 6), especially the note "The tour of building sites - or tools and vision" (n ° 178).

666 (\*\*) This is, at least, if it is considered "goal" that is displayed before the world ( "the Advancement of Science", say)

or even one, can not, which would consist in the expansion of a prestige, by the accumulation of forcing works the esteem and admiration. Yet it seems to me that even this "benefit" -There is incidental, before continuing satisfactions by the most powerful occult forces, those to which my friend has chosen to give dominion over his being.

667 (\*) This sub-note to the note "This work..." (N ° 171 (ii)) is exclusively mathematical nature. It can be omitted a player who would not feel encouraged to understand so little, in mathematical terms, the work of Zoghman Mebkhout and "Yoga 3-Modules" as new "theory of factors" in the cohomology theory of varieties. The pages

The following can be considered as a short introduction to yoga, or "philosophy Mebkhout" located in Under a conceptual baggage and a crystal clear overview. It was clear to me from the year 1966.

This vision was obscured systematically and essentially complete by my cohomologistes Deligne students, Berthe lot, Illusie Verdier, who had been the primary repositories. The only written record that remains is the text of my talks in 1966 IHES " **Crystals and the De Rham cohomology of schemes** ", notes by I. Coates & O. Jussila, exposed in Ten on étale schemes, North Holland Pub. Co. (1968). This presentation contains however, the technical point of view, all starting ideas of crystalline cohomology. Apart from the work of Mebkhout it does not appear that no progress really crucial was made at the conceptual level (or other) - on the contrary, I see a staggering decline compared my ideas of the sixties. They unfortunately appear very piecemeal, or between the lines, in the city exposed - the most important gap, here as elsewhere, is the absence of any explicit reference to the problematic tick coefficients De Rham, and a formalism of six operations (and bidualité) to establish for such coefficients (x). I could see that Mebkhout yet familiar than anyone else with my written work on cohomology (and one of my students), completely unaware that original problem (until a further two years) - and it seems to me that perspective "substrate" mathematical (and disregarding psychic non-intellectual factors), it was up still its main handicap.

Thereafter, I will refer to the quoted statement of 1966 [Crystals].

(X) (June 16) for a correction, see note b. p. (\*\*) page 990.

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18.5. THE FOUR OPERATIONS (a corpse)

(A) **The album "De Rham coefficients"**

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(4 May and 19-20 May) I recall that for analytical space p. 988 smooth complex tick, the term  $3 \times$  (or simply 3), the ring beam (more specifically, C-algebras) complex analytical differential operators on X. A first crucial fact, being evident dence by Sato, is that this is a ring beam **coherent**. A second fact tautological nature nonetheless crucial too, is that the O Class  $x$ -modules locally free, which we take as morphisms not the only morphisms  $O_x$ -linéaires but the differential operators between such modules, plunges as a **full subcategory** (but a priori functor **contravariant**) in that of 3-modules locally free by the contrafoncteur 668 (\*\*)

$$F \mapsto \text{Hom}_{O_x}(F, 3_d)$$

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$$- \text{Opdiff} \rightarrow (F, O_x)$$

(1)

$3$  where  $d$  denotes  $3$ , provided with its three-module structure induced by the canonical three-module structure **to right**, which switches with the operations

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$3$  left on itself (which the second member p. 989

(1) 3-Module). This fully faithful functor also induces an (anti-) **equivalence** between sub full categories formed free modules. It does not admit a canonical almost inverse functor "Switching to the restriction to an open" - that's why the first contrafoncteur envisaged is probably not (usually) an equivalence. If C (C like "crystal", see below) refers to 3-Module locally free (or even free, so be it), we can associate certainly a beam dependent functorially dec :

$$C \mapsto \text{Hom}_3(C, O_x)$$

(2)

Here we have a contravariant functor, which might seem to provide "the" natural candidate for a functor quasi-inverse of (1). The problem is that this beam (2) is not provided with a natural way to structure  $O_x$ -Module, but only a structure of  $C_x$ -Module (where  $C_x$  is the constant beam of X defined by the field of complex C). When C is from a  $O_x$ -Module locally free F by contrafoncteur (1), then (2) is canonically isomorphic to the underlying C-vector beam F.

The functor (1) is extended (as any additive functor) complex categories: it transforms a complex differential operators on X (the regular direction) in a complex of  $3 \times$ -modules locally free, and the (contradictory) X functor thus obtained is of course fully faithful (for differential morphisms between complex differential operators in the first complex category). It is in this sense that can be said that 3-Modules complexes (locally available components) " **generalized** " complex

of differential operators on  $X$ .

The view of the 3-Modules complex has the decisive advantage on the complex operators differential to be inserted directly into yoga (first developed in my 1955 article "On some points homological algebra" <sup>669</sup> (\*) ) Module complex on a ringed space and thus especially in that of **derived classes** (which I had emerged in the years following the cited article). The crucial concept of "**quasi-isomorphism**" does not appear to the naked eye when one adopts the view of morphisms differential between complex when she becomes manifest through the 3-Modules complex associated. So more than a **generalization** from the perspective of complex differential operators, introduced the viewpoint by Mebkhout <sup>670</sup> (\*\*) represents a **critical relaxation** :

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thanks to this point p. 990

<sup>668</sup> (\*\*) The isomorphism written here is  $\epsilon \circ u \mapsto \theta$  where  $\epsilon: 3 \rightarrow \mathcal{O}_X$  is "increasing"  $\theta \rightarrow \theta(1)$ .

<sup>669</sup> (\*) In Tohoku Mathematical Journal, 9 (1957) p. 121-138.

<sup>670</sup> (\*\*) (8 June) should be read here: introduced by Mebkhout in grothendieckienne range for the purposes of a new theory coefficients. It is understood that "the view of 3-Modules" is due to Sato, but used in any optical different.

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of view, and because of him only that the complex differential operators can be used now as "coefficients" for a new cohomology theory, with all the richness of intuitions attached to it. If I establish a parallel between the theory of De Rham coefficients, and the coefficients  $l$ -adic (which was also a major source of inspiration for Mebkhout in the development of his philosophy), I would say that this first step of **conceptual**, not a "childish", akin to that I had done (in 1958) by introducing the concept of beam spreads (containing the germ of the idea unifying critical of **topos**). In the same analogy, the "theorem of God" (we will remind more below) is similar to the base change theorem for a proper map in étale, who summer (1963) **the** first major theorem to the start of the étale, leading in space a few weeks to a situation of "control" almost complete on the cohomological tool spreads. The similar work in the context of 3-Modules (or more generally in the crystalline framework), to arrive at a mastery of the "crystalline cohomology" (or "De Rham" in a broad sense that I saw such a theory since the sixties) - that work remains to be done, for seven years the first big breakthrough was finally accomplished by Zoghman Mebkhout.

The new category introduced by Mebkhout coefficients, which "contains" (explicit meaning in note "The work of...", N° 171 (ii)) both "analytically Building discrete coefficients", and the coefficients consistent introduced by Serre (systematized by me in a cohomological theory of "coherent coefficients" <sup>671</sup> (\*) ) is that formed complexes of 3-modules cohomology beam **coherent** (as that 3-Modules), seen as full subcategory

$D^*$

$\text{coh}(X, 3 \otimes X)$  or  $\text{Cris}^*$

$\text{coh}(X)$

(3)

the usual derived category  $D^*(X \otimes X)$ . If one confines oneself to complex bounded cohomology (formant the full subcategory  $\text{Cree}$

$b$

$\text{coh}(X)$ ), such a "factor" is represented **locally** by a complex of free 3-modules of finite type in any degree, and bounded degrees; or too, which basically amounts to Similarly, a complex of differential operators in bounded degree. <sup>672</sup>

When working with the derived classes, there is of course necessary to replace the basic functors p. 991

(1) and (2) by total derivatives functor

$F \mapsto \text{Rhom}_{\mathcal{O}_X}(F, 3 \otimes X)$ ,  $C \mapsto \text{Rhom}_3(C, \mathcal{O}_X)$ .

(4)

If one seeks functors **covariates** similar in nature to these two functors, firstly falls on the functor "scalar expansion" (designated  $N$  in the note cited):

$F \mapsto 3 \otimes_{\mathcal{O}_X} F$ ,

(5)

(tensor product total), where the tensor product still used the structure  $\mathcal{O}_X$ -Module right of

$3$  ie.  $3$  of, <sup>673</sup> (\*) This functor  $F$  has the disadvantage, compared with (1), not to prolong the morphisms

<sup>671</sup> (\*) This is the formalism of six operations and bidualité, I developed the coherent framework in the second half fifties.

<sup>672</sup> (\*\*) (June 16 - see end of note (\*) page 988). Mebkhout just made me observe that this is not entirely accurate - this problem is addressed in loc. cit. 1.5 d) (p. 312). Mebkhout explicitly refers to it in his work "Poincaré duality"

(Seminar "singularities" of Paris VII, 1977-1979), in the last three lines of section 4.4 (duality theorem relating to the -modules).

673 (\*) It is known that  $\mathcal{O}_X$  is flat as  $\mathcal{O}_X$ -Module right or left (seen immediately on the canonical filtration  $\mathcal{O}_X$ , and the known form of the associated graduated. . . ). As a result, the tensor product "total" in (5) is actually a tensor product  
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$F \rightarrow F$  between arguments that are only differential operators (rather than linear). The second functor (4), it must look like a contrafoncteur

Cris \*

$$\text{coh}(X) \rightarrow D^*(X, \mathcal{O}_X)$$

also admits a "during" significant covariate, given by

$$C \mapsto \text{Rhom}_3(\mathcal{O}_X, C)$$

dfn

$$DR = (C) \text{ ("De Rham complex" associated with } C),$$

(6)

where the second member is explicit indeed a complex type Rham, through resolution canonical called "Spencer"  $W_X$ , by  $\mathcal{O}_X$ -modules locally free finitely generated. (This resolution is deduced from Ordinarily Rham complex, taking the  $\mathcal{O}_X$ -modules associated with the functor (1).) In terms crystalline (which will be explained below), the functor DR explicit as to the total derived functor function  $C \mapsto \text{Hom}_3(\mathcal{O}_X, C)$ , associating with each  $\mathcal{O}_X$ -Modules ("or" crystal) the C-vector beam formed its "horizontal" sections (of variable open). This is an operation **local nature**. The good concept (global) "**integration**" (or **object global cohomology**) for a "coefficient" C (ie a  $\mathcal{O}_X$ -module or as complex) here is not the usual functor

$$\text{Gamma-ray}_X(C) \simeq \text{Rhom}_3(X, \mathcal{O}_X, C)$$

but the functor (which is familiar to me as functor **total crystalline cohomology**) Total derivative functor "horizontal sections (global)"  $C \mapsto \text{Hom}_3(\mathcal{O}_X, C)$ ; I note this total derived by gamma-ray  $\text{cries}(C)$ , so p. 992 we tautological isomorphisms

$$\text{Gamma-ray}_{\text{cry}}(C)$$

dfn

$$= \text{Rhom}_3(\mathcal{O}_X, C) \simeq \text{gamma-ray}_X(CD(C)),$$

(7)

ie the crystalline cohomology  $C_X$  is obtained by taking the cohomology (global) normal complex De Rham partner.

Can be defined in Cris \*

$\text{coh}(X)$  a **Dualising functor**, resulting in a bidualité theorem, the model of those I discussed in the context (commutative) first coherent, discrete (spreads) then. I the will note D (as in the contexts cited):

D: Cris \*

$$\text{coh}(X) \approx$$

$$\rightarrow \text{Cris}^*$$

$$\text{coh}(X).$$

(8)

This is an anti-equivalence essentially involution (ie it has an isomorphism of bidualité, in functorial C:

$$C \simeq D(D(X)).$$

(9)

(9). This functor transforms (by composition) the contrafoncteurs (1) and (2) functors covariant. The simple fact to remember is that if C and C are "duals" of each other, then the De Rham complex (6) one is identified with the "co-Rham" (2) of the other (10)

$$\text{Rhom}_3(\mathcal{O}_X, C) \simeq \text{Rhom}_3(C, \mathcal{O}_X), \text{ and vice versa.}$$

(10)

On the complex of differential operators, this transaction D is expressed (a "shift" n close on the steps) ordinary.

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by passing the complex differential operators "Assistant" of Hom components  $\mathcal{O}_X(F, \omega_X)$  obtained in taking forward assistants term traders. Thus, the functor Dualising for  $\mathcal{O}_X$ -modules is compatible

familiar with the functor Dualising in duality of Serre,

$$F \mapsto \text{Hom}_{\mathcal{O}_X}(F, \omega_X) \simeq E \otimes_{\mathcal{O}_X} \omega_X \quad (F \text{ } \mathcal{O}_X\text{-Module loc. Lib. Finitely})$$

(11)

where  $\omega_X$  denotes the "modulus Dualising" differential forms of highest degree on  $X$ . We will be careful that the functor De Rham

DR:  $D^*$

$$\text{coh}(X, 3) \rightarrow D^*(X, C),$$

does not switch to generally dualisants functors (taking into the second category the functor  $\text{Rhom}_C(-, C_X)$ ). But it is a deep theorem Mebkhout (that everyone uses without citing per-sounds good and safe as if it were a simple sorite) as arguments **holonomic**, so for the functor induced

$$\text{Cris}^*(X)_{\text{hol}} \rightarrow \text{Cons}^*(X, C) \leftarrow \text{Cris}^*(X, C)$$

there is switching to dualisants functor. I "remember" not by the condition of **holonomy**, and confine myself to p. 993

report that a 3-module complex is holonomic sss its cohomology sheaves are 3-Modules

holonomic, and this is a natural condition of **local**  $X$ , and more, "**algebraic**". On the other hand,

Theorem constructability Kashiwara (the latter had stated for a **module** holonomic at a time when he nor anyone - except Mebkhout - was working with derived categories. . . ) Implies that the restriction the functor Rham complex holonomic carried well in  $\text{Cons}^*(X, C)$ . By introducing the concept of **regularity** of Mebkhout also local and "algebraic" Nature 674 (\*), we find the "good functor God "(aka Mebkhout)

$$m: \text{Cris}^*(X)_{\text{hol reg}}$$

$\approx$

$$\rightarrow \text{Cons}^*(X, C)$$

(12)

which, this time, is an **equivalence** (as we saw in the note "This work...", n ° 171 (ii)), which is therefore compatible with natural functors Dualising. It's almost the opposite functor (13)

$$M: \text{Cons}^*(X) \approx$$

$$\leftarrow \text{Cris}^*(X)_{\text{hol reg}} \leftarrow \text{Cris}^*$$

$\text{coh}(X)$

(13)

which allows to consider the category of "buildable discrete coefficients" ( $C$ -vector)  $X$ , as a full subcategory of  $D^*(X, 3)$  and specifically  $D^*$

$$\text{cons}(X, 3) = \text{Cris}^*$$

$\text{coh}(X)$ , we will inter-

pret sometimes as a category of "crystalline" coefficients.

(May 19) For now, we can say that we have described three "languages" or "views" different, such as many "photos" different, the same reality, or (essentially) a "same" type coefficients "called" coefficients Rham": there the view  $C$ -vector bundles and complex such (shooting "topological") with a condition of "analytical constructability" 675 (\*\*), acting a finiteness condition (essential, especially in order to write the type Riemann theorems Roch involving "characteristics of Euler-Poincare and groups Grothendieck" Suitable). he p. 994

674 (\*) I recall that the original definition of Mebkhout regularity was transcendent nature. For translation "purely ment algebraic", I refer to the presentation provided on the coefficients De Rham (style Mebkhout "style or" Deligne ") in volume 3 Reflections.

675 (\*\*) I remember that a  $C$ -vector bundle on an analytic space  $X$  is called "analytically building" if the neighborhood each point, he admits a composition series whose successive factors are of the form  $i_! (F)$ , where  $i: Y \rightarrow X$  is the inclusion of an analytical subspace  $Y = Z \setminus X T$  (with  $T \subset Z$  analytically two closed subspaces of  $X$ ), and  $F$  a  $C$ -beam locally free finitely (or "local  $C$ -vector system") to  $Y$ .

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is the view "complex of differential operators", with holonomy conditions and regularity taking the place of the conditions of constructability. And there is the view "3-Modules complex", with consistency conditions, Holonomy and due to the key. The second "picture" (taken under the angle "analysis ") is attractive, because it is we intelligible words" classic "and objects that we watch, learn complex differential operators, seem to "dimensions" reasonable, then the 3-Modules even coherent (starting 3) itself!), appear excessive when the look with the glasses " $\mathcal{O}_X$ -modules." Technically speaking, however, these provide a picture more complete. Indeed, while it is "clear" that locally each complex 3-cohomology modules



coherent and bounded degree (say) may be represented by a complex of differential operators via (1), is unlikely that this is also the case globally, if we do X draconian assumptions (gender "Stein manifold" or, in the algebraic framework, an assumption of quasi-projectivity) 676 (\*). The "picture" 1 has the advantage of keeping a sense when X is not supposed smooth, but is a space analytic any complex. By cons, as such, photos 2 and 3 are just as reasonable the assumption of smoothness. One can certainly still define a ring beam  $3 \times$  without assumption of smoothness on X, and there are still a tautological dictionary between complex differential operators (Component  $O_X$ -modules locally free) complex of 3-Modules (locally available components), but  $3 \times$  (It seems) ceases to be coherent, too bad! There is probably unlikely that a "theorem of God" could be reached in the particular case, on the model of the one known in the smooth. Obviously other except that need pictures of the type 2 or 3 in the singular case also, as the photo  $n \circ 1$  is **transcendent nature** : by tracing the naively in terms of Zariski topology or spread for a variety algebraic, one would find the "coefficients" too specific to be used (for these topologies are too coarse, based on the transcendental topology). Photos 2 and 3, by cons, restricted to p. 995 begin the field of vision "smooth", keep a sense "abstract" algebraic geometry (on a body because. zero, say, to begin with), making (for me) principal charm. This means that it is necessary to make enlargements, so that the singular varieties are included in the scope of vision.

It did not seem concerned Mebkhout who soucis- other well when I asked him, his immediate thought was this. Suppose X plunges into a smooth variety X as sub analytical closed space. While the category  $\text{Cons}^*(X, C)$  can be interpreted as the full subcategory  $\text{Cons}^*(X, C)$  consisting of objects whose restriction to  $U = X - X$  is zero (ie the support objects in X "). But it can also be interpreted by the theorem of God in terms of pictures 2 or 3, as the category of "coefficients de Rham - Mebkhout" on X whose restriction to U is zero. He must be easy to check a priori (remaining within the context of "coefficients De Rham - Mebkhout", ie the one Pictures of 2.3), that this category to equivalence set itself single isomorphism is Independent of the "lissification" chosen X X. I made myself full of things like that, and I want to believe that it works. If on the other hand is not X "lissifiable", never mind (Mebkhout said), we will "Make cohomological descent" to reconstitute a global category from these local pieces or introduce the "site lissifications" to open X, and work on it. There are chances that Can deal indeed, but instead of a "site lissifiant" (improvised Mebkhout for the purposes of 676 (\*)) Of course, nothing prevents to build a "class derived" from the category of complex differential operators X and differential morphisms between such complexes in "reversing" formally "quasi-isomorphisms" (defined transition to complexes corresponding 3-Modules). Be found (presumably) a subcategory **full**  $\text{Cris}^* \text{coh}(X)$ , but not the entire category probably, in the absence of assumptions like "Stein" or "projective X" (or only quasi-projective in the algebraic case). 769

reply in a conversation that remained platonic) site that strikes me as highly redundant, why not work with the crystalline site, which has proven itself (even if it has been forgotten, it would seem, with all related by those who were my students. . .)? And all the more so that it was clear to I, in the year 1966 when I cleared startup ideas lens yoga, that future "coefficients De Rham "should be expressed precisely in terms crystalline! This brings me out of my drawer bottoms a photo that has had time to amass the pous-IESO, the poor - yet once blew it, it appears to me as new, and perfect clarity. This is also one of the first things to what I have thought, writing last year (even before he meets the Burial. . .) Note "My orphans" ( $n \circ 46$ ), feeling obscurely that it was time someone speaks with respect about things that deserve respect. . . Moreover, since Mebkhout told me about 3-modules (1980 - God knows that I was not "connected" then!), I could not p. 996 help but think of it as the "crystal" instead, and use the words "3-Modules" and "crystal" (from  $O_X$ -modules) synonymously with (of course) a marked preference for the second. I therefore promised to the fourth photo, picture "crystal". Suppose first X smooth. To give 3-Module F on X is the same as give an  $O_X$ -Module with a structure addi-mentary, which can be expressed in various ways equivalent. One, the tautological, that is to say "extends" the operations  $O_X$  on the Abelian beam F in an X operation of the Ring  $3 \times$  (which contains  $O_X$ ). As  $3 \times$  is generated by  $O_X$  and the additive subbeam derivations, we see that it is the even to give F a so-called " **integrable connection** ", ie a law, every  $\xi$  derivation over an open U-X, associates a " $\xi$ -bypass"  $\cdot \theta_\xi F$ , linearly  $\xi$ , and so compatible with the operation "hook" to lead 677 (\*). We can say that this is a kind of structure "Differential" F of order 1.

Because that is characteristic zero <sup>678</sup> (\*\*), this structure can be interpreted also as a structural ture richer, a differential structure of infinite order, which I have called a " **layering** " on F (where F takes the name " **laminated module** "). One way to express stratification, is like a " **given lowering infinitesimal infinite order** " F (relative to the morphism  $X \rightarrow \text{a point}$ ), or more precisely ment, such as a given isomorphism, above the formal completion of  $X \times X$  along the diagonal, p. 997

between the two mirror images of F (by both canonical projections  $pr_1$  and  $pr_2$ ) isomorphism identity extends on the diagonal, and satisfies further to a "condition transitivity" suitable. Passing an integrable connection to a "data infinitesimal descent" (or laminated) represents a new idea - and "trivial" as all the new ideas that I "had the honor of dis- vvir! This however takes its strength once re-interpreted in terms of the concept of **crystal modules** . It shows that the structure in question on F also returns to the data, for any "neighborhood infinitesimal "U an open U-X, a **prolongation**  $F_U, F|_U \rightarrow U$  (in short, F" grows "above infinitesimal neighborhoods, such a "crystal" - crystal modules, in this case, but there are crystals of all kinds. . . ) - this extension behaving the way we guess for the concept of restriction <sup>677</sup> (\*) It is also, of course, a compatibility condition for the restriction to open.

<sup>678</sup> (\*\*) may, in the following, to overcome any event feature (as part of a smooth outline for, say) replacing the completed formal  $X \times_S X$  along the diagonal, by the formal completed "to S divided powers". it also conduit / O to a beam of  $\mathfrak{X}$ -Module F on X, replacing the pro-beam  $P_\infty(F)$  of its "main parts infinite order ", by" main parties divided powers (of infinite order) ". From dual side, this means replacing the sheaf of rings  $\mathfrak{X}/S$  of differential operators on (which is not consistent even if S is Noetherian) by the sheaf of rings "enveloping" relative derivations  $O_X \otimes S$  (which, what assures me Mebkhout would be consistent!). This is, in fact, the conceptual background for the coefficients De Rham, which will extend one of the Mebkhout 3-Modules, especially for the development of a theory of the coefficients De Rham for type schemes over Z. 770

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an open  $V \subset U$  and morphisms between neighborhoods infinitesimal (or "thickening")  $U, U$  of a even  $U$  (morphisms inducing the identity on  $U$ , of course).

Interest lens perspective is that the objects to be studied (3-Modules) can be interpreted as bundles of "ordinary" Modules on a suitable site <sup>679</sup> (\*), annealed in **local rings commutative** , namely the "crystalline site" formed by the thickened  $U$  various open  $U$  of  $X$  (the beam crystalline structural  $U$  being simply  $\mapsto \rightarrow \Gamma(U, O_U)$ ). Therefore, we have all the arsenal of insights Geometric associated with such a situation. A remarkable relationship I discovered in 1966 and then stunned me, is that the cohomology of the crystalline site (or topos of the lens corresponding to it) to coefficients in the structural beams (or more generally, with coefficients in  $F$ , at least when  $F$  is coherent  $O_X$ ), is identified with the **cohomology Rham**  $X$  (with coefficients in  $F$ , in this case, ie ordinary hypercohomology  $X$  with coefficients in  $CD(F)$ ). That was the start of the cohomology crystalline <sup>680</sup> (\*\*).

Thus we have a perfect dictionary, explained extensively in my presentations 1966 already mentioned <sup>681</sup> (\*), p. 998 between four types of objects  $X$ , or four types of structure on a  $O_X$ -Module:

(  
 $\left\{ \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$   
 $\left\{ \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$   
 3-Modules  
 $O_X$ -modules integrable connection  
 laminates modules (descent data infinite order infinitesimal)  
 crystals of  $O_X$ -modules  
 (Cr)

This dictionary is valid without any restrictions on the type or consistency quasicohérence  $F$ . Note However, if we compare the extremes

3-O Modules  $\Leftrightarrow$  crystal  $\mathfrak{X}$ -modules

natural concepts of "consistency" in either context **does not match** . The structural beam tural lens is consistent but coherent modules on the crystalline ringed topos exactly match the 3-modules that are consistent **as  $O_X$ -modules** , in which case they are even free finitely.

The category they form is canonically equivalent by the functor "scalar expansion" on  $C_X \rightarrow O_X$  , to the category of beam  $C_X$ -modules locally free, ie that of " **local systems**

$C$ -vector "X. This is so for such items, five possible descriptions (or five" photos "

by counting four of the table (Cr) above)! But these are "coefficients" excessively kind

Special <sup>682</sup> (\*\*), among those (De Rham - Mebkhout) of interest.

Let us return instead to the four pictures of the table (Cr) above, and see what happens when we do not suppose  $X$  more smooth. The four types of objects considered keeping a sense. It seems the other, both pre-

miers do not form important categories - rather, all three  $X$ -modules and all  $O_X$ -modules connection integrable is encountered naturally, such as "having a geometrical sense", "come" (In an obvious sense) laminated modules, which can also still be interpreted as crystals rate of  $O_X$ -modules, as in the case smooth 683 (\*\*).

679 (\*) We will be careful we do not find **all** the modules beams on the crystalline site, but only those who meet a simple additional condition (beams called "special" in [Crystals])

680 (\*\*) Again, startup ideas are so "trivial" it's really not worth bothering the little, when we spent fifteen years of his life, after, to develop a little bit (and forget the rest...).

681 (\*) See discussion [Crystals], quoted in the first footnote to this page sub-notes (Note (\*) page 988).

682 (\*\*) In fact, this is 3-consistent, of course (I had missed in the sixties) here the notion of finitude important.

683 (\*\*\*) this assertion was made hastily, and is false as is. For it to become true, replace the "site 771

I admit that without it some thought, I do not very much shows the exact relationship for  $X$  plunged p. 999

in smooth  $X$  (say) between crystals and crystal  $X$  (and this even when  $X$  itself is smooth) 684 (\*).

What is certain is that the crystalline site, or better, the lens  $\text{topos } X_{\text{cris}}$ , with its corrugated structure depends of the analytic space  $X$  covariant way, ie if  $f: X' \rightarrow X$  is a morphism between analytical spaces ticks, we deduce

$$f_{\text{cris}} : X_{\text{cris}} \rightarrow X'_{\text{cris}} ;$$

where such a functor "direct image" for Modules bundles on these ringed topos. We would like understand this operation (in the case of a closed immersion  $X' \hookrightarrow X$ , in particular) and to include what condition a crystal is transformed into crystal. It would also, in the case of a closed immersion, that this functor is exact. The idea here is this: if  $F$  is an object of the derived class  $D^*(X_{\text{cris}}, O_{X_{\text{cris}}})$   $F$  and its image by the total derivative functor  $f_{\text{cris},*}$ , and assuming more  $X$  smooth, provided that either  $F$  regular holonomic **should not depend on the chosen immersion of  $X$  into a space (smooth  $X$** . If it p. 1000

is so, then we will define the category of crystalline coefficients De Rham - Mebkhout  $X$  as Full subcategory (the derived class) as defined by the above condition (apparently on local  $X$ ).

Thus, the module work foundations should be done for twenty years and who apparently always remains days to do (about the fundamental operations on crystalline modules), we can say that in If  $X$  is any analytic space (not necessarily smooth), it remains **two** pictures (instead of four) to describe the "coefficient de Rham" which we have: there  $\text{Cons}^*(X, C)$  varietur, and is the category (which for the moment remains hypothetical, and as such I can not see yet 685 (\*\*)) of coefficients "De Rham - Mebkhout"  $\text{DRM}^*(X)$ , for which I have to hazard a definition principle. The  $\text{Cons}^*(X, C)$  category, the description offers no problem of the transcendent perspective, **disappears** However, as soon as one passes algebraic context. This makes clear the need to identify a good definition  $\text{DRM}^*(X)$ , which keeps a sense in this context. And it is clear to me also that the right "frame" for this photo, which suddenly seems (at first glance at least) the only one left is the one formed by the crystalline modules 686 (\*\*).

lens "formed by all the infinitesimal thickening of open  $X$ , for the subsite (called "laminating Site ") formed by those locally admit a retraction  $X$  (automatically condition satisfied when  $X$  is smooth). When we gives a laminate modulus  $F$  of  $X$ , its inverse image by such retraction **does not depend**, single isomorphism of the chosen shrinkage, resulting in a "canonical extension"  $F$  above the intended thickening. It is thus seen that when  $X$  is not smooth, a crystal structure  $F$  is "richer" than a lamination, since it can extend  $F$  (ie to "grow") above infinitesimal neighborhoods **whatsoever** to open  $X$ , including (and this is something of particular importance), above the infinitesimal neighborhoods of all kinds of  $X$ , immersed in an ambient space **smooth**. It is, in fact, that the new concept the most crucial and fruitful, between that of laminate module and that of crystal modules, is the latter. It is she who is called to dominate the theory of coefficients De Rham. I "remember" about it as clean and smooth outline for  $Z$  on  $X$  cohomology De Rham on  $Z \times X$  (as in the transcendental context qu'@gébrique...) is "not only" provided with a lamination, but rather a crystalline structure through the "grow up" on any infinitesimal neighborhood.

This is a **fact** crucial math, that Deligne had also forgotten even before my departure in 1969 when he described coefficients type Rham in terms of procohérents modules **laminated** instead of the highest crystalline version ie in terms of **crystals** of procohérents modules. It must be said that my name was attached less known to the notion laminate module (so natural that one would swear she must go back to the last century), that the notion of crystal Modules, much less "traditional" look. On this subject the reflections in "... And hinders" (subnote n° 171 (viii)).

684 (\*) (May 26) The situation has clarified for me with the introduction of the concept of co-crystal, in which it is refers to D) below.

685 (\*) I'd down allusion to a "fifth picture", which is much clearer to me now, to capture

"Good" coefficient Rham by a purely algebraic language crystalline words, keeping a sense without hypotheses smoothness. This photo is taken at an angle somehow "dual" to that of the photo-Mebkhout Rham.

686 (\*\*\*) I call " **crystal module** " X modules a beam on the lens ringed topos X **cries** . We can therefore consider

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I recognize also that even if X is smooth, I get confused not so much in the des-Mebkhout scription of coefficients "De Rham" in terms of the functor-good God it respects not natural multiplicative structures: the contra-functor Mebkhout which will be discussed in (b) that (it seems) it is compatible <sup>687 (\*\*\*)</sup>. A fortiori, this functor does not switch "to the six operations." Linen-that tuition (attaches to the coefficient Mebkhout seems very different in nature, at first sight, <sup>p. 1001</sup> that which is attached to discrete coefficients. That is, from a certain point of view, an advantage - we have two photos taken from radically different angles! It just makes it harder, than used to sous.un watch these angles well recognize themselves in the picture taken under the other. In fact, in addition to four photos already reviewed (for "De Rham coefficients", I hear) there is a **fifth** <sup>688 (\*)</sup> that I kept in reserve: it is that of Deligne, to pro-modules shots laminate <sup>689 (\*\*)</sup>. It has the advantage of "stick" very close to the intuition of constructible discrete bundles: item "zero degree" corresponds to an object of the same type, product concepts and tensor image inverse correspond by equivalence Deligne; So it-be the same for all six operations (which may indeed be described in terms of these two). Moreover, the passage of operation "coefficients De Rham - Deligne"  $DRD^*(X)$ , those of De Rham - Mebkhout  $DRM^*(X)$  seems to me principle particularly well understood, in terms of operations ( "O<sub>X</sub>-dualité") on  $O_X$ -modules (at least of all, for X smooth) - I have already alluded in a previous footnote on page <sup>690 (\*\*\*)</sup>. I have So the impression here of being on land in both solid and familiar, which should enable me to recognize myself, as soon as I take leisure. I thought even sketch (in this note the view of Deligne, and to <sup>p. 1002</sup> the link with that of Mebkhout and formalism outlined in my talks already mentioned in 1966. But this subnote begins to be long, and becomes increasingly digress! So I prefer to refer the thing Reflections Volume 3, where I think also to the description of "good" coefficient Rham (Deligne style, or Mebkhout choice) on type schemes over Z.

**(b) Form of God** (5 May and 21 May) I would come back here on the description of the functor Mebkhout (also called "the good Lord")

M:  $Cons^*(X, C) \rightarrow Cris^*$

$coh(X)$  (

dfn

=  $D^*$

$coh(X, 3X)$

(1)

modules crystals as special cases of crystalline modules.

<sup>687 (\*\*\*)</sup> This "it seems" is a somewhat casual way (almost like "new style" ...) to retract a beautiful theorem, due always the same unknown service (but more recent vintage, I grew to understand, that of God). He implies for example, two analytically closed subspaces Y and Z of K, the following formula on the local cohomology, obviously too good to be true even (and yet...)

$\gamma\text{-ray algebra}$

$Y \cap Z (O_X) \approx \gamma\text{-ray alg}$

$Y (O_X)$

The

$\otimes \gamma\text{-ray algebra}$

$Z (Y_X)$ ,

some fine gentlemen will also pocket one of those fine mornings, I bet, like "if they had always known" - in  
Meanwhile the award to the best of them. . .

<sup>688 (\*)</sup> So, I did better than keep the promise of the title of this note "The five pictures": I have actually highlighted **two series** of five photos, the first describing the only "coefficients De Rham," and the second crystalline coefficients general.

<sup>689 (\*\*)</sup> As reported in a previous note b. p. (Note <sup>(\*\*\*)</sup> page 998), this photo of De Rham - Deligne taken with an "objective" a little biased (for indeed reasons beyond the competence of the worker manufacturer). he is needed to retouch, and also enlarge, making her the scope of the zero characteristic. This will be done in the Volume 3 of Thoughts, where dear former students can come in all pumped ease the "unnecessary detail" and other "digressions

technical "they have not had time to find themselves, for almost twenty years since I left them to fend themselves with a splendid subject in hand. . .

<sup>690 (\*\*\*)</sup> This "previous note of b. P." turned meantime in part (c) of this note "The five pictures."

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wherein  $X$  is a smooth complex analytic space. As we said in the note "This work..." (N ° 171 (ii)) is there a functor profound nature, defined as quasi-inverse functor restriction functor De Rham DR subcategory full  $\text{DRM}^*(X)$  (the "coefficient de Rham-Mebkhout" on  $X$ )

$$\begin{aligned} & \text{Cris}^* \\ & \text{coh}(X), \\ & m = \text{CD} \mid \text{DRM}^*(X) \text{DRM}^*(X) \\ & \text{dfn} \\ & = \text{Cris}^*(X) \text{ hol reg} \rightarrow \text{Cons}^*(X, C) \end{aligned}$$

(2) which happens to be an equivalence ("theorem of God"). In fact, get a description Mebkhout remarkable direct function of  $M^\infty$ , deduce the functor  $M$  by the functor  $i$  "scalar expansion" by the homomorphism of the Rings

$$3 \times 3 \rightarrow \infty$$

$$X$$

$$(3) \quad 3 \text{ where } \infty$$

$X$  (or  $3^\infty$ ) denotes the ring of "infinite order differential operators  $X$ " that is, (by definition) that of  $(C\text{-}O$  endomorphism beam  $x$ , seen as a cluster of complex topological vector spaces.

It is known that  $3^\infty$  is faithfully flat left and right on  $3$ , so that the total derived functor of functor extension of the Rings

$$i \text{Cris}^*(X) = D(X, 3) \rightarrow D(X, 3) \rightarrow D(X, 3^\infty)$$

$$\text{dfn} \\ \text{Cree} = \infty(X)$$

(4) is explicit by an ordinary tensor product. Note that it does not know whether the Ring  $3^\infty$  is consistent but apparently it is on. We define the full subcategory

$$\text{Cris}^* \\ \infty(X) \text{ hol} \leftrightarrow \text{Cris}^*$$

$\infty(X)$  3-Modules complexes which are "holonomic", for the condition to derive locally (the functor  $i$ ) 3-Modules  $C$  complex which is holonomic. (The result of the double theorem of God, recalled Below, you can then take the same  $C$  both holonomic and regular, ie a "coefficient (De Rham - p. 1003

Mebkhout "and this determines  $C \times X$  on any single isomorphism...) We consider the functor  $M^\infty = i$   $M$ , fitting into the commutative diagram

$$\begin{array}{ccc} \text{Cons}^*(X, C) & & \\ M^\infty & & \\ (( & & \\ \text{QQQQQQQQQQQQ} & & \\ M & & \\ \text{uullllllllllll} & & \end{array}$$

$$(5) \quad \text{DRM}^*(X)$$

$$i$$

$$// \text{Cris}^*$$

$$\infty(X) \text{ hol} . \\ = \text{Cris}^*(X) \text{ hol reg}$$

It is (or rather, the unknown worker proves...) That the functor  $M^\infty$  is also an equivalence categories (so  $i$  also). It is available also as quasi-inverse functor  $m^\infty$ , like "From Rham "m like, set to  $\text{Cris}^*$

$\infty(X) \text{ hol}$  . To describe the functor  $M^\infty$ , it is more convenient to describe the contrafoncteur

$$. \text{delta } \infty \\ \text{dfn}$$

$$M = \infty D = D \infty M^\infty = i(R) = i(DM),$$

(6) where  $D$  is the functor Dualising mentioned in  $\text{Cons}^* \text{DRM}^*$  and  $D^\infty$ , the functor Dualising similar that exists in  $\text{Cris}^*$

$\infty(X) \text{ hol}$ , (and even in  $\text{Cree}^*$

$\infty$  coh (X)). (NB The three functors involved

in (5) switches to dualisants functor.) The quasi-inverse  $\cdot\delta$  of  $\cdot\delta$  is given by the formula

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analogous to (6)

$\cdot\delta$

dfn

$$Dm = \infty m = \infty D$$

(7)

We then find the expression of Mebkhout  $\cdot\delta$ ,  $\cdot\delta$  by the following two formulas, a symmetry remarkable:

$$\{\cdot\delta(F) = \text{Rhom}_c(F, O_X)\}$$

$$\cdot\delta(C) = \text{Rhom}_3(C, O_X)$$

(8)

Note that in the first of these formulas, the second member inherits a 3- $\infty$ -structure, thanks to 3 operations  $\infty$ ; on the second argument  $O_X$ , while in the second formula, the second member is simply interpreted as a C-vector bundles complex. The second of these formulas, up there "for memory" is also essentially tautological, and simply said that the functor  $\delta$  associated with the complex of 3  $\infty$ -modules C the complex differential operators (of infinite order) "assistant" of that associated with C (by the functor  $DR = \text{De Rham}$ ) - this complex being interpreted as complex C-vector bundles. (That one is good and a constructible cohomology sheaves complex equivalent to constructability Kashiwara theorem.)

(This is a deep theorem by cons, the first functor  $\cdot\delta$  transforms Building beams p. 1004 (complexes) 3  $\infty$ -modules that are holonomic. The only finiteness theorem implied by this result 691 (\*) (Not to mention Holonomy) is already in itself a remarkable new result. The thing even more extraordinary however, **is that both functors are quasi-inverse to each other**. formally, thus resembles bidualité relations, which can be expressed either in the category  $\text{Cons}^*$  or in  $\text{Cris}$  category \*

$\infty(X)$  hol - except that the contrafoncteurs "dualisants" (expressed in both cases as a  $\text{Rhom}_\infty(-, O_X)$ ) interconnect two categories **different**. It is this formal analogy Mebkhout led to call the theorem that says isomorphy

$$\cdot\delta \cdot\delta \approx \text{id}$$

in  $\text{Cree}^*$

$\infty(X)$  coh

(9)

the "**bidualité theorem**" for complex 3  $\infty$ -modules (terminology also likely to cause confusing). This relationship, plus the fact that the functor  $\cdot\delta$  is fully faithful (or more precisely, that  $\cdot\delta$  is an assistant, something he includes in his statement bidualité theorem) was obtained by Mebkhout in 1977, before the theorem of complete good God. The theorem says "of bidualité" So basically means, (like "my" bidualité theorem, which it derived) a complex of 3  $\infty$ -modules **holonomic** can **reconstruct**, as an object of a derived class with the knowledge of the complex of operators Differential (infinite order) associated, given simply as a C-vector beams complex (in category suitable derivative); and more specifically, it can be reconstituted by the formula explicit inversion (8) (first formula). A fortiori, a morphism between complex 3  $\infty$ -modules holonomic is a quasi-isomorphism if and only if the morphism corresponding to complex differential operators (infinite order) is in the naive sense (ie induces an isomorphism on cohomology sheaves) 692 (\*\*).

691 (\*) This result implies finiteness e.g. locally on X, the complex  $\text{Rhom}_\infty(F, O_X)$  is isomorphic (in the category derivative) to a complex of 3  $\infty$ -modules which is locally free finitely generated in each stage, and its modules cohomology derived (locally), by extension of scalars, coherent three-Modules. In fact, one can even assume these holonomic and regular.

692 (\*\*) (26 May) In fact (as I pointed out below, start (c)) Mebkhout prove this result, even outside any condition Holonomy, in the equivalent form: if the complex differential operators associated with a complex of 3  $\infty$ -modules is almost zero, it is the same of the latter (and ditto for 3-modules).

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(Mebkhout bidualité The theorem is somehow "half" of the theorem of God

p. 1005

(for the 3  $\infty$ -modules), when it is taken in its strongest form, saying that the functors

(8) are substantially inverses of each other. This is the main result of the thesis Mebkhout submitted in January 1980. But this "half" in itself, is already a new result and (as far as I know) fully unexpected. It is a typical result, bridging the gap between the ideas of Sato and mine, but in the optick my long program formulated through "continuous" or "differential" (and with a view derived categories), the "discrete coefficients." As such, it seems to me that this result totally escapes me, by his spirit and his inspiration to the problems of the Japanese school of analysis. theorem constructability Kashiwara seems to have represented a "next", and not the starting point of a new theory of the coefficients. As publications for the period between 1976 and 1980 evidenced no possibility of doubt, Mebkhout was only then to develop such a philosophy. Mebkhout had spoken of his results to Kashiwara, visiting Paris, in January 1978, while ve-born to finish writing his thesis. At the request of Kashiwara, candid Mebkhout all happy to have finally found someone who looks like interested in what he has to say, sent him to Princeton chapter III all warm - that in which among other things says theorem "of bidualité". That was in February 1978. Three years later, the same result figure (with a semblance of proof do) in a famous article Kawai Kashiwara- 693 (\*). It was renamed "theorem reconstruction" for the occasion, and without any allusion to some Zoghman Mebkhout. Moreover, it was also the memorable year of the Symposium Perverts - the glorious year of a "new style" 694 (\*\*) won high hand (and without encountering any resistance. . . ), This part of mathematics, among all, where I had once wont feel at home

...  
**(c) The fifth picture ( "pro")** (May 21) The "bidualité theorem" (9) is 1977. prou-  
 p. 1006

ver the other half of the "theorem of God" for the  $3 \infty$  -modules, returning once to prove that the functor  $\cdot \text{delta} \infty$  is essentially surjective, the first difficulty was to prove that in Cons for  $F^*$ , and by defining the complex of  $3 \infty$  -modules  $C = \cdot \text{delta} \infty (F)$  by the first formula (8), that it could be obtained via the functor  $i$ , at least locally on  $X$ , using 3-Modules complex (holonomic, regular). A priori, according to the ideas of Mebkhout (ie following the dual theorem of God, implying that the functor  $i$  in (5) is an equivalence), the latter was to be one single almost quasi-isomorphism. I have not tried to understand how Mebkhout finally unraveled in his thesis for

build this 3-Module. It seems to me that the situation should clarify here, using the idea of Deligne procohérent the beam associated with a building C-vector bundle  $F$  695 (\*). This idea was developed by him in the context of varieties **algebraic**  $X$ , but must adapt mutatis mutandis the analytical cases, provided perhaps work "locally" on  $X$ , or each compact  $X$ . fais-CWater procohérent associated to  $F$ , which is (at least on each compact  $K X$ ) an inverse system  $(F_i)$  of coherent beams (defined near  $K$ ) can be defined quite simply as the beam

693 (\*) Mr. Kashiwara, T. Kawai, is holonomic Systems of micro-differential equations, III Systems with regular singularities, Publ. RIMS 17, 813-979 (1981). The "reconstruction theorem" looted in Mebkhout is s. 4 of this long (received in November 1980). Labor main result is a weakened variant that the functor  $i$  in (5) is an equivalence categories. So this is an immediate corollary of the theory (geometric) of Mebkhout result that these authors obtained by analytical way (independent of Mebkhout). See for details the sub-note "The Mafia" n ° 171 (ii) Part (b): "First trouble - or Kaidis overseas Pacific".

694 (\*\*) See, on this "new style" (which Kashiwara and Hotta are eminent rivals overseas Pacific) Note "The congratulations - or: the new style" (n ° 169 9 ).

695 (\*) This is the idea that he had developed in his seminar at IHES 1969-70, then left behind. On this subject the sub-note "... and hinders" (n ° 171 (viii)).

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Pro-represents the functor

$$G \mapsto \rightarrow \text{Hom } c (F, G)$$

on the category of  $O_X$  -Module consistent  $G_X$  (in the vicinity of  $K \dots$ ), wherein functor, being exact for left, is pro-representable. For example, if  $F$  is the constant beam  $C_Y$  on an analytical subspace closed  $Y_X$ , "extended by zero" on all  $X$ , we find the profaisceau consisting of  $O_{X_n}$  where  $X$  are the infinitesimal neighborhoods of  $Y$  in  $X$ . (NB The inverse limit of the inverse system is completed formal  $O_X$  along  $Y$ .) It is found (returning to the general case) that the pro-beam  $(F_i)$  is provided with a canonical stratification 696 (\*\*). The idea of Deligne is that the "**functor ( Deligne "** from the category p. 1007 Plot of C-vector bundles on  $X$  to the category of pro-coherent beams laminates, is **fully faithful**, and therefore allows to interpret the first category (which is transcendent nature) in Under a full subcategory of the category of laminates pro-coherent beams. The latter has a purely algebraic sense, and the full subcategory in question can also be defined (more or less tautological 697 (\*)), in terms purely algebraic also. This is the category that I will note  $\text{DRD}^*(X)$  ou  $\text{Del}^*(X)$ ,

(10)

which is the " **fifth picture** ," I did not want to explain yesterday <sup>698 (\*\*)</sup>. I think I remember elsewhere Deligne that had bothered to develop its interpretation (and full fidelity previous statement) of so that it passes the derived classes (at a time when he had not yet been decided by my cohomologists students unanimous Deligne head to sell off the last), and this is the version "category derived "I denoted by (10), of course.

That said, the "algebraic part" in  $\text{Rhom}_c(F, \mathcal{O}_X)$  must be able to define very natural as an inductive limit (in a suitable sense) of  $\text{Rhom}_{\mathcal{O}_X}(F_i, \mathcal{O}_X)$  - in particular (from the beams cohomology) discloses canonical arrows

$$\begin{aligned} \lim_{\rightarrow} \\ \text{Ex d} \\ \mathcal{O}_X(F_i, \mathcal{O}_X) &\rightarrow \text{Ex d} \\ C_X(F, \mathcal{O}_X) \\ (\forall d \in \mathbb{Z}) \\ (11) \end{aligned}$$

Using lamination to the pro-object  $(F_i)$  and tautological laminating the second argument  $\mathcal{O}_X$ , it must be possible to define on the first member (11) ie a lamination of 3-module structure, such that (11) is compatible with the homomorphism of operators Rings (corresponding p. 1008  $\mathbb{Z} \rightarrow \infty$ ). That said, the theorem of God to Mebkhout must clarify whether, saying that (11) identifies the second member  $\mathbb{Z}$  to  $\infty$ -Module derived from the first by extension of scalars <sup>699 (\*)</sup> - which implies in particular that the arrow is an **inclusion**. Thus, the left member must view as being a kind of **part "Algebraic"** (or "**meromorphic**") in the right hand member (which itself is liable <sup>696 (\*\*)</sup> The concept of stratification for a pro-Module is defined in the same way as a module - the description in the notes of the previous day (part (a)) in principle applies whenever we have a notion of "relative" (such as modules, pro-Module concerning pattern etc.) assuming a concept of "image" reverse, ie resulting in a "fibered category" on the category to "Variety" on which we work. . . Attention if you will  $(F_i)$  is a pro-Module, a stratification of it can not generally be described in terms of a "compatible" system of stratification of  $F_i$ . - the envisaged objects are likely many More generally the pro-objects in the category of laminated modules.

<sup>697 (\*)</sup> "Tautological" at least in terms of the known dictionary (first released by Deligne) between C-vector bundle locally constant (or "local systems") in the additional  $Y - Z$  a  $Z$  divider in an analytic space  $Y$ , and coherent modules laminated to  $Y - Z$  which are "scheduled" (in the sense of Deligne) along  $Z$ .

<sup>698 (\*\*)</sup> Finally, this explanation (called a "circular"! ) Is not given either here, at least not immediately. She however, will be given away (page 1011). We will focus the notation (10) refers to the variant "derived categories."

<sup>699 (\*)</sup> In addition, of course, the first member (11) (consistent with the philosophy of Mebkhout) must be 3-Module **coherent**, **holonomic** and **regular**.

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"Transcendent").

The general situation is illuminated considerably on the previous specific example,  $F = i^*(C_Y)$ , where  $i: Y \rightarrow X$  is the inclusion of a closed analytic subspace of  $X$ . Then the second member (11) is a local cohomology beam supports in  $y$  - an invariant **transcendental**, while the first member

$$\begin{aligned} \lim_{\rightarrow} \\ \text{not} \\ \text{Ex d} \\ \mathcal{O}_X(\mathcal{O}_{X_n}, Y_X), \end{aligned}$$

is the well-known phrase that I had introduced to the local cohomology in the schematic part. The fiber of the beam at a point  $x \in Y$  is something other than the local cohomology on the spectrum  $X_x \times \mathcal{O}_{X,x}$ , the structural beam supports in the "trace"  $Y_x \times Y_X$ .

We see in this example how the idea of Deligne is close to those I had developed on the theme of local cohomology debut sixty years <sup>700 (\*\*)</sup>. Still, the main theme the work of Mebkhout between 1972 and 1976 was precisely to study the arrow (11) in this crucial case

$$\begin{aligned} \lim_{\rightarrow} \\ \text{not} \\ \text{Ex d} \\ \mathcal{O}_X(\mathcal{O}_{X_n}, X) \\ \text{dfn} \\ = H_d \\ Y(\mathcal{O}_X) \text{ alg} \rightarrow H_d \end{aligned}$$



Y (Y X).

(12)

It proves in this case the relationship announced earlier, and more (something I had sometimes failed to include in statement) as the first member (12) 3-Module **coherent** and even, holonomic and regular. From there, the analogous statement for (11) must be an immediate consequence by unscrewing 701 (\*\*), (including the p. 1009

case F, instead of building a C-vector beam is a complex in  $\text{Cons}^*(X, C)$ . The only cents, except the shaped construction of the functor Deligne, is in the definition of  $\text{Rhom}_{O_X}$  a laminates promodules complex values in a complex laminated modules ie in a complex 3-modules (in this case,  $O_X$ ), as 3-modules of complex (and as an object of a category derived).

Modulo this grain of salt, we thus find a description of all that has simple and conceptual, the functor of God M "algebraic" (as opposed to the functor of God M  $\infty$  "transcendent"), or rather Associated contrafoncteur  $\Delta$  and its near opposite  $\delta$

$$\Delta = MD = DM$$

$$\delta = TND = Dm,$$

(13)

by a double-paraphrase formula (8). But to write it, using the equity method of Deligne

$$\text{Del: } \text{Cons}^*(X, C) \approx$$

$$\rightarrow \text{DRD}^*(X)$$

(14)

let@just watch the corresponding functors  $\Delta, \delta$  between  $\text{DRD}^*(X)$  and  $\text{DRM}^*(X)$ , where the signs are supposed to remember that we will work (the "Building" side) with **pro-objects**. We then find the formulas outstanding (morally contained in (8), but this time linking coefficients "of algebraic nature @

700 (\*\*). It will appear lower than the idea of Deligne is also closely linked to that I had introduced in 1966 (in [Crystals]): for any complex differential operators, I consider his "formal"  $P \infty (L \cdot)$  as a pro-module complex laminates or, more preferably, as defining a **crystalline complex**, whose crystalline cohomology (global) is identified with the cohomology (overall)  $L \cdot$ .

701 (\*\*\*) (May 22) I am a bit bright here! The "basic types" of Plot C-beams are more general than only  $C_Y$ , (But it is true that the proof of the general theorem uses the same technique as the particular case of 1976) 778

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one and the other, and this by formulas "of algebraic nature @s well):

$$\{\Delta(C) = \text{Rhom}_{O_X}(C, O_X)$$

$$\delta(C) = \text{Rhom}_{O_X}(C, O_X).$$

(15)

So here we have twice the "same" formula, with the only difference that C is here a complex of faisceaux laminated pro-consistent (or what amounts to the same 702 (\*), a complex crystal module pro-consistent), while C is a 3-modules (can be seen morally as a complex

$O_X$ -modules ind-coherent layered, or, as a crystal ind-coherent Modules). It@the

"Even" essentially functor that passes from one to another, namely, the "ordinary functor Dualising" (Coherent), old friend fifties. . . It is "obvious", of course, that it must share pro-objects and ind-objects (even move to the inductive limit in these...).

(Of course, there is a foundation working to do, to give a precise meaning to these formulas - a labor p 1010.

the type made by Deligne in his famous seminar scuttled, or Jouanolou in his famous thesis

also scuttled (everyone quotes from the Symposium Pervert, and no one held in his

hands. . .). This is a job, I@ sure, that may be a little long, but essentially "sorital". The

"hard" part is contained in the theorem of God to Mebkhout completed by formulas Mebkhout

(8) called (improperly maybe) Formula "bidualité". Their algebraic translation by against, claiming

both functor (15) are quasi-inverse to each other, is indeed (morally) "the" theorem

Ordinary bidualité coefficients for  $O_X$ -cohérents, made ind pro-sauce and laminations in the key

(Who must "pass" without problems in the Dualising functor).

The correspondence between the two types of objects dual displays are perfect (without any work

foundations thrown in!) in terms of complex differential operators. (In this duality, moreover, the

provided Holonomy (and a fortiori, that of regularity) plays no role.) In such a complex  $L \cdot$ . The functor

$F \mapsto \text{Hom}_{O_X}(F, \mathcal{O}_d)$  (contravariant) proposed yesterday (in (a) (1)), combines a 3-modules complex

components locally free of finite type or C. On the other hand, the "formalization" of the  $L$  complex  $\cdot$ , in

passing the main parts of order  $P$  infinity  $\infty (L_i)$  (regarded as laminates promodules) provides a

Complex  $C = P \infty (L \cdot)$  laminates pro-modules. That said, we see that these two complexes correspond

by formulas (15), in which here, obviously the Rohm reduces to Hom. (Just check this

to-one duality for components  $L_i$ , and it is then reduced to the fact more or less tautological

that "continuous" linear homomorphisms  $P^\infty(L_i) \rightarrow O_X$  correspond exactly, as are linear homomorphisms  $L_i \rightarrow \mathbb{3}$ , the differential operators  $L_i \rightarrow O_X$ , using respectively the differential operator "universal" (infinite order)  $L_i \rightarrow P^\infty(L_i)$ , and "increase"  $\mathbb{3} \rightarrow O_X$  Data by  $\theta \mapsto \theta(1)$ . As at least locally on  $X$ , any object  $\text{Cris}^*$   $\text{coh}(X)$ , (ie all complex 3-coherent cohomology modules) are described by means of a complex of differential operators  $L_\cdot$ , we can consider that for all practical purposes, this case provides a perfect grip on duality (15) between the two types of coefficients provided to speculate 3-consistency and "3-pro-coherence" Suitable  $C$  and  $C$ , "dual" from one another. It is therefore sufficient to develop the "sorite" which I refers, limiting, or  $C$  "pro" side, complexes procohérents beams (laminates, p. 1011 locally, can be described (almost quasi-isomorphism) as a  $P^\infty(L_\cdot)$ . Compared to the original approach of Deligne, the fact that pro-coherent modules and complexes of such it introduces, can be realized locally by a complex of differential operators, is also a **entirely unexpected phenomenon**, brought by the theory Mebkhout. It seems to me essentially equitable 702 (\*) See Note b. p. (\*\*\*) page 1006, about this translation. 779

worth 703 (\*) at Mebkhout theorem mentioned above (from 1976, even before the demonstration Theorem of God), for 3-consistency of  $H$  beams  $d$

$Y(O_X)$  alg (which appear in (12) above above). This is a deep theorem culmination of four years of work, and using the full force Hironaka resolution of singularities (not counting the courage of the workers who cleared and proved to against the general indifference). The consequence 703 (\*) I just mentioned is a deep relationship coefficients between De Rham (as I glimpsed them from 1966) and complex operators differentials, relationship I had nothing planned (or Deligne either, when he developed his first approach to the coefficients De Rham). As to the condition holonomy and regularity of the complex of differential operators considered, it must be equivalent (a posteriori, thanks to providential theorem God) to the condition of "finite" (more "regular") Deligne (I sometimes failed to explain, in intro- duisant category  $\text{DRD}^*(X) = \text{Del}^*(X)$ ). It is this: cohomology pro-beams  $P^\infty(L_\cdot)$  to "unscrew" locally by composition series, such that successive factors can be décroissent (via the functor Deligne) by local  $C$ -vector system has subspaces  $Y - Z$   $X$  (wherein  $Z \subset Y \subset X$  are analytical closed subspaces of  $X$ ). To complete to give this criterion a appearance "algebraic", just replace the local system of  $C$ -vector by a beam **coherent** laminate  $Y - Z$ , subject to the condition that the connection which expresses lamination (NB we can assume  $Y - Z$  smooth) or "regular" in the vicinity of  $Z$  in the sense of Deligne 704 (\*\*). (NB. The combined pro-beam is obtained by by growing the (crystal that has  $YZ = T$  above infinitesimal neighborhoods of  $T$ , and "crushing" p. 1012

along  $Z$ , to have coherent beams everywhere, not just in the complement of  $Z$  ...)

**(d) crystals and co-crystals - fully faithful?** When one assumes more  $X$  smooth, it remains, to describe "De Rham coefficients"  $X$ , in addition to the "photo" of transcendent nature  $\text{Cons}^*(X, C)$  the two "pictures" (crystalline in nature and the other one)  $\text{DRM}^*(X)$  or  $\text{Del}^*(X)$ , which they have meaning purely algebraic. I outlined yesterday (in (a)) a principle of definition for  $\text{DRM}^*(X)$ , and today for category  $\text{DRD}^*(X)$ . It is the latter which now gives me perfectly intelligible.

As I reported yesterday (see (a), note b. P. (\*\*\*) page 998), applicable here refine the perspective of Pro-laminated modules, by the crystal pro-Modules (pro-coherent) 705 (\*). The only problem that remains even with this view, the sorite "pro" that require to develop, sorite who (according to my modest experience in such matters) may take prohibitive dimensions! These crystals promodules, associating, to each infinitesimal thickening  $U$  of an open  $U-X$ , a pro-coherent Module  $U$ , "Compatibly with mirror images" to morphisms  $U \rightarrow U$  thickened, can not not even be interpreted as pro-beams on the crystalline site (or what amounts to the same, on the topos 703 (\*) (May 26) Again, I am "a little lively", the result 1976 is not enough. Compare with commentary note b. p. (\*\*\*) 1008 page.

704 (\*\*) The regularity condition is introduced here naturally, given the equivalence of categories generated by Deligne between the local systems of  $C$ -vector  $Y - Z$ , and the fibers integrable connection on  $Y - Z$ , provided with a "Meromorphic structure" along  $Z$ , and regular connection along  $Z$ . This meromorphic structure (involving the possi- bility to extend the coherent Module  $Y - Z$  in a coherent Module  $Y$ , at least locally in the vicinity of each point  $Z$ ) was implied in the earlier description given.

I understand when one drops the regularity condition in the above condition (assuming simply given a meromorphic structure of  $E$  in the vicinity of  $Z$ , in order to associate a pro-coherent module on  $X$  entirely by the Deligne method), there is a description "cohomology" of the condition Holonomy. The definition is by Sato

way "microlocal" - I have never really learned yet, I admit. . .

705 (\*) (27 May) On reflection, I even difficult to believe that the theorem of Deligne Cons \* (X, C)

Del \* (X) is true for X

nonsmooth when Del \* (X) is defined as the Deligne without recourse to the crystalline site. This is perhaps to be realized he finally preferred to scuttle the whole theory, rather than consent to reintroduce the taboo site. . . (Compare note "... and hinders" n ° 171 (viii).)

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crystal  $X_{\text{cris}}$  )! So one can not a priori implement them known cohomological formalism beams Modules on topo (commutatively) annealed, such  $X_{\text{cris}}$  .

The temptation here to go to the projective limit of profaisceau we have on each thickened.

Thus we find crystalline Modules (otherwise crystals Modules), the "value" of each U has nothing coherent or quasi-coherent. The hope is that at least for (the type of pro- crystals p. 1013

Modules of interest (those in particular obtained by the functor Deligne) such crystal pro-modules can be **reconstituted** from crystalline Module C deduced by passage to the limit, taking on

each U thickening "envelope pro-coherent" C zariskien the beam  $U$  , (C restriction to open zariskiens U) 706 (\*). That seems to be the case at least for the pro-crystals associated modules

a coherent laminate module on a Y - Z as above, for example in the typical case where one takes the formal completed  $O_X$  along Y - Z and it extends by zero elsewhere (and ditto on the thickenings).

If my "hope" is justified, then the category DRD \* (X) coefficients De Rham - Deligne X could

be interpreted as a full subcategory of ordinary derived category  $D^*(X_{\text{cris}}, O_{X_{\text{cris}}})$  defined by terms like "finite" and "regularity" (themselves described in terms of unscrewing, as before

above) on the cohomology sheaves. It would be then a description amazingly simple, that

I also would have been able to give in 1966, if I had taken the leisure while continuing my crystal reflection. . .

This question "foundations" (if it is lawful to pass to the limit) clearly not depends on the question if X is smooth or not - it is not, is immersed in a smooth and X is reduced to the smooth case. If this

view (almost too good to be true!) walked indeed, then (in the smooth now) it

would place suddenly (I think) to interpret the formulas "of bidualité" (algebraic version) (15) as being  $Rhom_{O_X}$

**ordinary** without the burden of pro-issues (but by simply being careful

to carry the stratifications ...). A first test in this respect is as follows: if  $u: C_1 - C \rightarrow 2$  is

a morphism of complexes of 3-coherent cohomology modules, such that its image by the functor naive dualizing  $Rhom_{O_X}(-, O_X)$  is a quasi-isomorphism in it is the same for  $u$ ? But this means

(By an argument mapping-cylinder) to ask if 3-modules of complex coherent cohomology, as its "dual naive" is zero (as defined cat. derived, ie at zero cohomology sheaves) is itself

zero (the same direction). Or, if you have the complex of differential operators  $L$  . He is the same to say the associated 3-Modules complex is zero cohomology sheaves, or so be it for

complex "formalized"  $P^\infty(L \cdot)$  , seen this time not as a pro-beam complex, but as a

Complex ordinary beam (passing to  $\lim \leftarrow$  ). Mebkhout will surely be able to tell me. . .

((May 23) I even phoned Mebkhout last night - it@also good for a week or two I p 1014.

phone him almost every night for mathematical or historical questions - and in total, it@

a note of astronomical phone! But the apotheosis, which I loin me and I polishes for three weeks tightly packed, well worth it. . .

Still Zoghman that guaranteed me a result that the neighboring air "test question" on which I finished last night: if C in  $Cris^*$

coh is such that the complex operator  $L \cdot = DR(C)$  associated

is almost zero, then C is itself almost zero (analytical case). It was a complex homomorphism beams (C-vector), given by the "main parts of infinite order"

$L \cdot - P \rightarrow \infty(L \cdot)$  )

706 (\*) Speaking here of "zariskien" beam (as opposed to "crystalline" I liquorice surreptitiously in the schematic context. Readers who prefer the analytical context has rectified itself.

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hence homomorphisms

$$H_i(L \cdot) \rightarrow H_i(P^\infty(L \cdot))$$

( $i \in \mathbb{Z}$ )

(16)

the cohomology sheaves. You want to say that this homomorphism (16) is always injective, and

identifies the first member to the sub-beam sections "horizontal" of the second (which would be a kind functor of the accuracy of property "beam horizontal sections" on a pro-Modules category suitable laminates. . . ). The injectivity already imply that if the second member is zero, it is the same First, so if this is true for all i (and from what assures me Mebkhout) 3-Modules complex associated with L . is almost zero - what I wanted.

The injectivity in (16) also means that for a differential operator E

d

-  $\rightarrow F$ , and a section of F that f

at each point  $x \in X$  is "formally" in the image (passing to the completed local ring of the point), and such more than the "formal solution" (from equation  $(g) = f g$ ) can be made, for x variable, dependent analytically x - Equation locally then admits a solution. Mebkhout said he did not knowledge of such a result; yet the question is so natural that the answer should be well known!

To conclude the "five pictures," I would still come back here on both "crystal clear pictures"

one corresponding to the viewpoint of Mebkhout 3-modules, the other to the dual standpoint. He good

heard that one must work in the spirit of the derived classes - so a rendition of "crystalline" worthy

of the name should reflect that. So the two crystal photos are "fully faithful" if the

functor corresponding (ranging from Class D b

coh  $(X, 3)$  (say), to a suitable crystal category,

p. 1015

such that  $D_b(X_{\text{cris}}, O_{X_{\text{cris}}})$ , is itself fully faithful. I hope that that is the case, **without even bother with holonomy conditions and regularity** of the complex of 3-planned modules.

The simplest case is probably that of the photograph n ° 4, which is to interpret the category of

Modules 3-like crystals of modules, resulting in a total derivative functor (called "Grothendieck" - for

take the lead on fans of "unnecessary detail" and "technical digressions." . . ):

G:  $D^*$

coh  $(X, 3) \rightarrow D^*(X_{\text{cris}}, O_{X_{\text{cris}}})$ .

(17)

The crucial question here is whether this functor is fully faithful. It is only in this case the notation

$Cris^*$

coh  $(X)$  to the first member is fully justified - and with it, also, the lens view

in cohomology Rham (at least in this case, in the complex analytical framework, or part of

algebraic patterns on a body because. nothing). To prove the full fidelity, in algebraic geometry

say, we are reduced by standard arguments in case X is affine (or, in the analytic case, in case

a polydisk), and in case the two objects C, C envisaged in the first member (including the case of

compare Hom in both directions) are both equal to 3 itself, with just a shift of

degrees. (This reduction is no problem, at least assuming C, C bounded degrees, so by

merely  $D_b$

coh  $(X, 3)$ , which seems quite sufficient for applications) We are thus led to verify

finally formulas

$\Gamma(X, 3_X) \sim$

$\rightarrow \text{Hom}(G(3), G(3)), Ex i$

$O_{X_{\text{cris}}}$

$(X_{\text{cris}}; G(3), G(3)) = 0$  for  $i > 0$ .

(18)

(X affine, resp. Stein). I have not taken the time to check 707 (\*), but have little doubt that this is true.

I demonstrated something very similar, I think, in [Crystals] (1966) 708 (\*\*).

707 (\*) I apologize, most of my time, for over a year, having been absorbed to track prowess

some of those who were my students. . .

708 (\*\*) This is the result I have already alluded elsewhere, for a complex of differential operators L . on a diagram

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### 18.5. THE FOUR OPERATIONS (a corpse)

(As for the photo five, there are several different prints. Deligne The original drawing is in terms of p. 1016

laminates pro-coherent modules. The first major retouching, for the generalization to non-X

smooth, is to interpret the animals in question as **crystal** pro-ons. But it agrees

there in the gear (uninviting!) endless pro-pro-cohomological foundations of algebra - and

loses the benefit of direct intuition topossique attached to  $X_{\text{cris}}$ . Also I prefer (if possible) outright

take another photo, the same angle of view pretty much via a functor **contravariant**

(Said also "Grothendieck" to the wise...)

$G_o : D^*$

coh  $(X, 3) \text{ opp} \rightarrow D^*(X_{\text{cris}}, O_{X_{\text{cris}}})$ .

(19)

We can say that this is one that is derived from picture Deligne passing brutally limits beams projective on each infinitesimal thickening an open  $U$  of  $X$ . If  $C$  in the first member is associated (of contravariant way as in formula (1) (a)) to a complex differential operators  $L \cdot$ , Its image (19) is obtained by looking  $P^\infty(L \cdot)$  (The "formalize" the complex  $L \cdot$ ) As a complex laminates promodules (idea introduced in [Crystal]), or as a crystal complex pro-Modules, and passing to the projective limit on any thickening. Another way of saying this is at any  $O_X$ -Module locally free (e.g.)  $L_X$ , is associated with a lens module (which is not **not** a crystal modules, I believe), I note that  $P^\infty(L \cdot)$  cries, in an "obvious" certainly (and my Students have long forgotten), which module functorially depends on  $L$  with respect to operators differentials, and therefore passes to complex differential operators.

One either preceding description of the functor (19) remains also incomplete, in particular because an object of the first Member does not necessarily come on all  $X$ , a complex of operators differentials. I suppose we can give an intrinsic heuristic interpretation of this description, the formula

$$G \circ (C)$$

~

$$- \text{Rohm} \rightarrow O_{X_{\text{cris}}}$$

$$(G(C), O$$

$X_{\text{cris}}$

) (Wherein  $G$  defined in (17))

(20)

but did not check it is correct. By standard arguments, it still brings back here (to prove that natural boom (20), where  $C$  is associated as above in  $L \cdot$ , Is an iso) where  $C = 3$ , and then (20) reduces to the formulas (

p. 1017

$\text{Ex } i$

$O_{X_{\text{cris}}}$

$$(G(3, O_{X_{\text{cris}}}) = 0 \text{ for } i > 0,$$

(21)

which quite resemble (18).

The sense of full fidelity (19) is in any case quite clear and is reduced again by unscrewing (and as in (17)) where  $C = 3$ ,  $C = 3 [i]$  (degrees shift of  $i$ ), and then reduced to the formulas

$$\Gamma(X, 3) \simeq \text{Hom}(\mathcal{F}_i, \mathcal{F}_i), \text{Ext } i$$

$O_{X_{\text{cris}}}$

$$(X_{\text{cris}}; \mathcal{F}_i, \mathcal{F}_i) = 0 \text{ for } i > 0,$$

(18.1)

when we asked

$$\mathcal{F}_i = P^\infty(O_X)_{\text{cris}},$$

smooth on (or in the analytical framework ditto, surely), the hypercohomology "zariskienne"  $L \cdot$  is identified with the hypercohomology

crystalline logy its formalized  $P^\infty(L \cdot)$ . Actually, this statement relates more directly the arrow (19) "dual" (17), and

can also be expressed by saying that for  $C, C$ -3-Modules complexes coherent cohomology, the arrow

$$\text{Hom}(C, C) \rightarrow \text{Hom}(G \circ (C), G \circ (C))$$

is bijective, if  $C = O_X$  (which is not bad and allows all hopes...).

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which is a crystalline Algebra all it is remarkable on  $X$ . It is assumed (for the nullity of  $\text{Ex } i$  crystalline) that  $X$  is affine (resp. Stein).

Finally, what seemed to me yesterday "almost too good to be true," then I saw

even things through the photo Deligne suddenly beginning to look like everything he has reasonable there - a

Once things are written without the hassle of Holonomy terms (much less, regularity).

If God willing, and if no one else does the work for me before, I hope to get the thing

clear (and the validity of (21) and (18)) before the end of the year, with the part of Volume 3 of Reflections which will be devoted to the coefficients De Rham.

As I said, this is the picture five, the one who "sticks" as close to the topological intuition partnering with discrete coefficients, which is my preference. This is the heavy heart that I would learn that the formulas (22)

are false (although I would be less bored if this were the formulas (18), which, however, have

Air technically less screwed). This would show that he would return to the pro-perspective (the photo

Deligne retouched) - a perspective not so cheerful! Anyway, there's no doubt for me that

technical adjustments, we take indeed an excellent picture, including valid geometry

algebraic (and even on something as characteristic zero) and without hypothesis smoothness.

As for the photo four, whose loyalty is subject to the validity of (18), I confess again that I "Do not see much" still outside the smooth case (and in the smooth), and am not sure for X not smooth, crystal clear interpretation I have proposed march indeed such. It seems to me my endemic perplexities variance, for perspective Mebkhout 3-modules (and especially, my crystal interpretation of this view) are about to be solved by the introduction p. 1018

a dual notion that crystal, which I call **co-crystal**. It was as recently as yesterday that this feeling diffuse malaise that there was (for the "variance" of the 3-Modules by closed immersions) finally ended give birth to a "good idea" (in what seems to me, without really written anything yet). Sounds paste side "independent", as well as the notion of crystal (which is familiar to me) the "pro" side. On a smooth variety, two categories (crystals and co-crystals) are canonically equivalent (which is why I necessarily had tendency to confuse - it's excusable. . .), But it is the same for any X. The situation is quite analogous to what happens with the cohomology H ring . (X) and the cohomology group H . (X) or the ring Chow Ch . (X) and the Chow group Ch . (X), or Grothendieck ring (I apologize the odd. . .) K . (X) and the group of K Grothendieck . (X) (re-apology). There too long we confused the two types of objects when X is a variety (topological or algebraic etc. - depending on the case) smooth. It "explains" after the fact that the second term is provided in any case a structure the first module (the "cap" -produced - in the latter two cases it was introduced by an ancestor I dare not mention here. . .), And in the smooth case, we find that this module is free of rank 1 and provided with a canonical basis, which did confuse unintentionally with the ring (even more beautiful is heard).

Well so is the Cree categories

. (X) crystals Modules X, provided with a structure

"Ring" by the tensor product, and that Cris . (X) of the co-crystal modules, on which the previous

"Operates" by a cap-product, perfectly!

But it is time to end this long digression mathematical completely moved (I admit)

in scheduling a beautiful Funeral Ceremony. The interested reader to know what happened (bushy, it will

of course) will be reduced purchase volume 3 Reflections (if he complained his money), where an unrepentant deceased his confused account pursue "technical digressions" 709 (\*).

709 (\*) This time is it necessary to say, as a "collaborator" of another of my students, promoted long ago "father" of

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### 18.5. THE FOUR OPERATIONS (a corpse)

**(e) The ubiquity of God** (May 27) A "last" footnote page, added to the "Five Photos"

in extremis yesterday (before giving striking the first twelve notes of the Apotheosis), has taken yet "of prohibitive dimensions, "and I (finally continue" this long digression mathematical "by p. 1019 last (and short) section. Thus, "The Five Photos" will consist of the **five** sections (a) through (e) - as what all rounds and perfect will. . .

This is a comment on the true validity domain (alleged) the "theorem of God" of Mebkhout which far exceeds (in my opinion) the initial part of complex analytic spaces - not only by **philosophy** news he brings (and has now renewed the theme cohomological), but also in a technical sense.

Once interprets Building C-vector bundles on X (smooth), or in terms of Modules laminates procohérents (at Deligne) or (by passing the inverse limit on thickenings infinitesimal to open X) in terms of crystalline beams (Grothendieck), the "good of the theorem God "alias Mebkhout affirms the equivalence of two categories which, this time, are **one and the other** of nature "purely algebraic". In other words, this theorem is now taking a specific meaning in other contexts as complex analytical context: both the context of smooth patterns on a body (that there is not even be assumed characteristic zero - see this note b. p. (\*\*)) Page 996 upper ; by bus.  $p > 0$  the view "crystalline divided powers" is essential here) or varieties rigid-analytical of any characteristic or smooth patterns finitely Z (and so on...).

The "formal" part of the theorem of God concerns **all** the complex consistent 3-Modules, not only those who are holonomic, and said that the functor of God, revised and corrected by care ancestor (ie duality to the structural beam  $O_X$  essentially) is **fully faithful** of

Class  $D_{\text{coh}}(X, \mathcal{O}_X) = \text{Cris}^*$

$\text{coh}(X)$ , towards the envisaged class coefficients Coeff \* socket on the person (taste). When one takes things well, it should be more or less "sorital".

But in the finish category is defined, "by unscrewing" two remarkable full subcategories, that of "coefficients.holonomes" resp. that of the "regular holonomic coefficients" (as at the end of (c)) and in Note b. p. (\*\*)) page 1011). That said, the "Mebkhout theorem generalized" (as contemplated context) that he will have nothing but sorital certainly is surely deeper, say two things:

1. (The class  $\text{Coeff}^*$  hol of "coefficients" holonomic is in the image of the Creps category  $\text{coh}(X)$  by p. 1020 functor (fully faithful) "of Mebkhout-Grothendieck". (NB. Morally this functor is the functor of Mebkhout but looked on Cris  $\text{coh}(X)$  as a whole, and more "revised and corrected by care ancestor", for the purpose either  $\text{Coeff}^*$  that a purely algebraic sense...).

2. To characterize the inverse image  $\text{Coeff}^*$  hol and  $\text{Coeff}^*$  hol reg by conditions of "holonomy" and "Regularity" "microlocal" in terms of complex differential operators. For the latter (which for my sixties program may be relatively accessory) it has a characteristic zero Holonomy already provided all found. As for the regularity condition, it's time to see if the Japanese would not have just the right concept in their sleeves - but is not Mebkhout who teach me, as he has seen too much to want to hear about it. As for me who have not seen like him, it seems to me there **three aspects** different regularity, that complement each other:

1. Appearance "geometric" generated by Deligne by unscrewing in  $\text{Coeff}^*$  hol, By returning to the condition regularity for a "local system" (p. ex. connection to integrated fiber) in the vicinity of a divider crystals. . .

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singular.

2. Appearance "microlocal" or "Japanese" are making directly in terms of complex operators différentiels (?)

3. Appearance "cohomological" introduced by Mebkhout aspect that is currently understood (it I think) that in the complex analytic case. I have no idea if he has a chance to generalize analytic rigidly. The appearance  $3 \circ )$  will of course be crucial whenever it will establish a **comparison theorem** between cohomology "zariskienne" and cohomology "rigid", for an algebraic variety defined over a value body complete, and holonomic coefficients. To my great "variances program" of the sixties, it is on the aspect of "geometric" that is the most important aspect of all. What is important is to define a formalism of the six operations the  $\text{Coeff}^*$  Reg hol . If there is even one for the  $\text{Coeff}^*$  hol As Mebkhout seems to believe, great. But (if I am not mistaken) reasons (which I have before (else) will not give birth to p. 1021 coefficients both holonomic and regular. I return to the question 1, which admits as obvious variant a "Question 1" (smaller), with coefficient hol substituted  $\text{Coeff}^*$  Reg hol . Once proven the full fidelity of the functor Mebkhout-Grothendieck it is visibly reduced to this: one takes on a smooth submanifold (not necessarily closed)  $Y$  with  $X$ , a fiber integrable connection (or an F-crystal  $C$ - consistent, depending on the selected context...), with necessary additional regularity condition to Deligne for it (the point  $Y - Y$ ). The process of Deligne (possibly reviewed by the ancestor to go to crystalline context) allows us associating an object there  $\text{Coeff}^*$  (which by definition will even "holonomic" or "regular holonomic"). This Is the object in the image of the functor Mebkhout-Grothendieck? Or, what is the same, do locally on  $X$ , the object in question  $\text{Coeff}^*$  can be described by a complex differential operators  $X$ , by the patented process of the ancestor of passing to "formalize" said complex, performed or as complex to Deligne or as a crystalline complex? The answer to this question is in any case so (I believe) in the complex analytic case, and

in the case of smooth patterns on a characteristic zero without having to introduce the regularity condition. This is the "completely unexpected phenomenon, brought by the theory Mebkhout" I've already taken care of previously stressed (in (c), page 1011) <sup>710</sup> (\*). In the regular case (including "Infinitely"), it is essentially the theorem of God. In the general case, if I am not mistaken, this must result without tears what I called the "Holonomy cohomological criterion" (or "reciprocal: the Theorem constructability Kashiwara ") due to Mebkhout discussed in the following note " Three milestones - or innocence "(n ° 171 (x), see page 1028).

**b2. Three milestones - or innocence**

**Rating** 171 (x) ((5 May and 23 May) <sup>711</sup> (\*)) The philosophy Mebkhout has developed between 1972 and 1980 can p. 1022

<sup>710</sup> (\*) Emphasize such facts became nowadays, at least in the part of mathematics in question here, a real **work of public health** , at a time when almost all publications on cohomological theme, and all (I fear) of those that appear in the now prestigious names are written in such a way to **evade** precisely major key ideas that sustain all these texts, and to **blur** or **eradicate** r the role and origin of such crucial tool (old or new emerged) such nerve concept in such fertile idea. There is **corruption** intellectual (signs of deeper corruption ...) which runs today in our science and in view of everyone, which I have no knowledge for no other science at any other time in history.

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be summarized in **three main theorems** , all three closely related to the ideas I had developed in the fifties and sixties, but I (or anyone) had been able to foresee any <sup>712</sup> (\*\*).

The first main theorem is the main result of the work of Mebkhout between 1972 and 1976. It concerns bundles **local cohomology**  $H^i$

$Y(O_X)$  (concept introduced independently by Sato and I) of the beam

a structural analytic complex smooth variety  $X$ , stands in a closed analytical subspace  $Y$ .

Observation essential here that no one had thought to do before Mebkhout is that operations

ring  $\mathcal{D}_X^3$  of infinite order differential operators  $X$  <sup>713</sup> (\*\*\*) , as they (operate on the argument p. 1023

$O_X$  , also operate on these beams cohomology. Furthermore, under "zariskien" geometry

algebraic, I described these beams (late fifties?) as inductive limits

Ext beams

$i$   
 . This Mebkhout led by analogy, to introduce a "algebraic" part of the cohomology local, and a canonical arrow

$H^i$   
 $Y(O_X) \text{ alg}$

dfn  
 $= \lim$

$- \rightarrow$   
 not

Ext  $i$   
 $O_X(O_{X_n}, O_X) - \rightarrow H^i$

$Y(O_X)$   
 dfn

Ext  $= i$   
 $C_X(C_Y O_X)$ ,

(1)  
 where  $X_n$  denotes the infinitesimal  $n$ .ème vicinity of  $Y$  to  $X$ , and  $C_X, C_Y$  constant beam  $C$  on  $X$  resp.  $Y$

(The latter extended by zero  $X-Y$ ). The second essential observation is that this time the ring  $\mathcal{D}_X^3$  ordinary differential operators on  $X$  operates on the first member. It was well known that the kind of beams that got both the transcendent nature of RHS, which is on the left

of "algebraic" nature were quite prohibitive dimensions as  $O_X$ -modules - nothing consistent,

It's certain. It is also true that we felt (at least the algebraic side) there was still a

some type of "finite" or "cofinitude" in a sense that no one before has thought Mebkhout specify.

The remarkable theorem Mebkhout is that the first member is a  $\mathcal{D}_X^3$ -Module, and again, the second member (who had even more intractable air) is simply derived from the first by the change of the Rings

$\mathcal{D}_X^3 - \mathcal{D}_X^3 \rightarrow \infty$

As the second ring is known to be flat on the first, it implies also that (1)

injective. At the same time, saw the result of consistency / this can be considered a finiteness theorem

<sup>711</sup> (\*) This sub-note "The three milestones" comes from a footnote on page note "This work..." (N ° 171 (ii)). See the sign



referring placed towards the end of this note.

712 (\*\*) As I pointed out in the note "The absurd questions" (n ° 171 (vi)), yet I knew a long time  
Alternatively the theorem of global dual Mebkhout a diagram for clean and smooth on  $X/S$ , in terms of complex  
related to differential operators. Specifically, if  $L$  and  $L'$  are complexes such "Assistant" of each other, then  $f^*(L)$   
and  $Rf^*(L')$ , as objects of the derived class  $D(S, \mathcal{O}_S)$  are "perfect" complex (locally representable by  
Free Modules complex finitely bounded in degrees), and the duals of each other in the usual sense for complex  
perfect. In the case where  $S = \text{Spec}(C)$ , this theorem is more or less equivalent to that of Mebkhout (restricted to the case of a  
analytical and algebraic variety that is clean), with this important difference, however, was missing the point of view  
"Derived categories", to address complex of differential operators. Secondly and most importantly, I had no suspicion  
These complex (subject to appropriate conditions released by Mebkhout) form a perfect substitute for "coefficients  
discrete "(or coefficients Rham). It was clear to me, on the other hand, since 1966 at least, there had to be such  
substitute algebraically constructible C-vector coefficients, having a sense for patterns relating to characteristic  
any, and my crystal ideas were just a first approach in this direction. As discussed in [Crystals]  
(these are exposed cited in the previous note "The five photos (crystals and 3-modules", n ° 171 (ix)), the internal logic of  
my thoughts were crystal clear to me yet again brought in contact with the complex differential operators. I was then  
already close to the philosophy of Mebkhout. It was my cohomologistes students (especially Deligne Berthelot Illusie)  
are blocked by the syndrome of Burial, not to have cleared this philosophy soon thereafter. (Myself  
was then fully occupied with other tasks foundations, and had left the crystal theme care of my students.)  
713 (\*\*\*) For a definition of these operators, whose name is scary at first, but that result in a formalism in all  
point parallel to that of ordinary differential operators, see part (b) of the previous note "The five photos (crystals  
and 3-Modules)" (n ° 171 (ix)).  
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very strong on the second member (to which person before Mebkhout did not understand) - it is  
including finitely presented as  $3^\infty$ -Module (but perhaps not consistent, since we know whether  $3^\infty$   
itself is consistent).

The first case treated Mebkhout that of a normal crossing divisor, the subject of his thesis  
Postgraduate, passed in 1974. Already this case is not trivial, and of course, entirely new - the  
same question resolved by Mebkhout had never been seen. This case also proves to be the crucial event,  
which happens Mebkhout (successive approximation, of increasing generality) to return to 714 (\*), with blows  
resolution of singularities.

(The result that I just mentioned, by itself, seems to me such an extent that under conditions  
p. 1024

so slightly normal, they have earned their author international recognition. Also, the first  
critical cases treated by him already denoted an originality of vision that "normally" would have earned him the en-  
warm agement of those among their elders (like each of my former students, without exception) who were  
able to enjoy the flavor. Let©move. . .

In fact, in these four years, Mebkhout reaches a more detailed result yet as I  
just stated. It proves that 3-module he studies is not only consistent but more **holonomic**  
(a notion he found in Japanese school) and more **regular** 715 (\*) (in a sense that it defines ad hoc in  
Inspired by my comparison theorem for the cohomology of algebraic-analytical De Rham). Better  
again it proves that the building C-vector bundle starting  $C_Y$  (which falls within the definition of  
second member (1)) is **reconstructed** from the complex of  $3^\infty$ -modules  $\text{Rhom}_{3^\infty}(C_Y, \mathcal{O}_X) = C$ ,  
the extraordinary inversion formula:

$$C_Y = \text{Rhom}_{3^\infty}(C, \mathcal{O}_X).$$

(2)  
No one had ever dreamed of such a formula - and nobody will dream until J five years later  
when the power of philosophy reveals and gives also the signal for the burial, the co-  
sides of the ancestor of the one who had brought it. . . To dream, it would have not have buried philosophy  
Ancestors (with blows of derived classes, with or without Rohm underlined and other "unnecessary detail"...);  
and more, to appreciate a geometrical situation any innocuous and yet full of mystery (the coho-  
logy to local media in a divider normal crossing), and go **to the end** of the mystery. This  
"End" it is not yet in the splendid 1976 theorem that I have just described - but from that moment  
Mebkhout has a clear vision: the double "theorem of God", one for 3-holonomic modules  
regular, one for  $3^\infty$ -modules holonomic, and the double inversion formula (or "bidualité") which  
discussed previously 716 (\*\*). It is also the solution of a marvelous simplicity, the problem of  
the relationship between discrete coefficients (Building analytically) and "continuous" coefficients.  
(But I look. When he proved the theorem that is the first major milestone of his work and

p. 1025

philosophy, the "end" clearly perceived, yet it seems dizzily far. If he had found at  
him a competent and caring elder, and with a minimum of experience and mathematical acumen, it

would have disabused: obviously, he was already close, and the difficulty to overcome, as so often in the Working discovery (if not, always...), was more psychological than technical. But before

714 (\*) For the Mebkhout theorem on the local cohomology, see: Local cohomology of a hypersurface in Functions of several complex variables III, Lecture Notes in Mathematics n ° 670, p. 89-119, Springer-Verlag (1977), and Local Cohomology of analytic spaces, Publ. RIMS Kyoto Universe. 12, p. 247-256 (1977).

715 (\*) The original definition (transcendent) of Mebkhout regularity is recalled in the note "The work ..." (n ° 171 (ii)), Note b. p. (\*) page 950.

716 (\*\*\*) In the previous note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), part (b).

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### 18.5. THE FOUR OPERATIONS (a corpse)

embarking in pursuit of the infinitely distant, it tackles the global duality theorem - the one who was "Cap" the duality theorem known both for consistent coefficients as coefficients discreet. The underlying motivation, omnipresent in the work of Mebkhout that connects the two issues, the local and that of the global cohomology duality is the u feeling **not essential unity** between discrete coefficients and continuous coefficients. This was also my thread in my crystal approach 1966, which attempted to apprehend the "coefficient de Rham" (essentially discrete nature) in terms "continuous". . .

This is not the place to come back here on the statement of Mebkhout duality theorem 717 (\*). his demonstration ran into serious technical difficulties due to the transcendent context, it overcomes to blows cohomological descent techniques and nuclear EVT (techniques that my person did not most were foreign, although Mebkhout is the only one that still persists in the name ancestor. . .). From the point of view his philosophy of duality, this theorem is a milestone. If one keeps in mind, with Mebkhout applied to complex 3-holonomic modules it contains global duality for the coefficients dis-analytically constructible decrees 718 (\*\*), in addition to the consistent duality, we can say that already contains the seeds, too, all the philosophy of the 3-Modules to Mebkhout. Its scope, as soon as he told me about the first time in 1980 (the year after defending his thesis 719 (\*\*\*)), appeared to me as a thing obvious. I do not think I had the honor of inspiring work of comparable scope, no student working on my contact 720 (\*).

p. 1026

Mebkhout has also had great difficulty in getting published this theorem, which felt "the grothendieckeries" full nose. (The Annals of Mathematics the sacked him, making him understand that that kind of Things did not have the required level. It ended up looking the same, in Mathematica Scandinavica in 1982 721 (\*\*).) I think it was there his favorite subject, when he gave lectures on phi-phy-3 modules, but in a very different spirit from that of the Japanese. He told me that this theorem had every gift to amaze listeners or casual partners, except precisely in every time, those who are part of the establishment 722 (\*\*\*). This is one thing that comforts me. She 717 (\*) This statement is recalled in the note "This work..." (N ° 171 (ii)).

718 (\*\*\*) As Mebkhout establish its global duality theorem (1976), it has also not proven yet that all beam analytically Building C-vector derived from a 3-modules complex. But he had no doubts about it.

719 (\*\*\*) See Note "from beyond the grave Love" (n ° 78).

720 (\*) I am thinking especially of the students who prepared a thesis with me. If Deligne is at hand, as he spends his thesis after my departure, and without also pronounce my name, while the inspiration from his work (in cohomology Hodge-Deligne) came to him from my issue of "coefficients" of all kinds, which also provided a formalism the "coefficient Hodge." The work of Deligne is a first step in this direction, much more fragmented than accomplished by Mebkhout in the (closely related to that of Hodge) of "De Rham coefficients." It is true that Mebkhout faced with severe disabilities due to indifference and scorn of his elders, was not distressed by cons the burial syndrome that has paralyzed my students. (On this subject see the note "... And hinders" n ° 171 (viii).)

721 (\*\*\*) theorems overall duality for coherent 3-modules, Mathematica Scandinavica 50 (1982) p. 25-53. See also "Poincaré duality" in seminar on singularities of Paris VII (Pub. No ° 7), 1977-1979, and especially "The Poincaré-Serre Verdier duality" in Proceedings of the Conf. Of Algebraic Geometry, Copenhagen (1978), Lecture Notes in Mathematics n ° 732, p. 398-418, Springer Verlag (1979). The introduction to both of these presentations, especially the second, represent a sketch of philosophy given by Mebkhout at a time when he was the only one to be the custodian and the lawyer.

722 (\*\*\*) (May 24) This overlaps well with my own observations. Apparently the man situation with a view predisposes such sufficiency, for whom "nothing is good enough for it deigns to rejoice." I do not know if these are the rule throughout the scientific world today, even, forever. That was my big chance of being welcomed in my early days in an environment where such a spirit of complacency does not exist - yet.

He had come on tiptoe, over the years, settling permanently in some and in others, little by little, without anyone us (apart Chevalley only...) noticing. Everything seemed the same as before - and yet everything was different already. It was already like a thin layer of dust on us, covering the original freshness of things. I was touched by this dust, like the others. And today, when I am again faced with one of those who were students,

shows that this spirit of sufficiency force-fed, which tarnishes the beauty of all things, however beautiful she did not become General in the mathematical community. It occurs primarily (if not exclusively) in the upper echelons, where I had ample opportunity to actually get to know the past ten years. . .

(It should complete this duality theorem overall by the result already mentioned local in nature, p. 1027

too deep, saying that the functor natural Dualising for 3-modules complexes, bundles coherent cohomology, wherein complex holonomic in holonomic complex transforms (and ditto for regular holonomic complex) is more compatible with these functor De Rham CD ( "complex of differential operators associated ", regarded as complex C-vector bundles to cohomology Building gy) for the functor natural Dualising I had brought on them 723 (\*). this compatibility is obviously an essential ingredient of Mebkhout duality formalism for understanding meaning its overall duality theorem. For some reason that escapes me, he calls "duality theorem local " 724 (\*\*). This deep theorem, as the famous " match "(called" Riemann-Hilbert " when we deign to name) is processed by "everyone" (Verdier and Deligne head) as a thing "Well known" that would go without saying, and above all without ever naming some unknown (which "everyone" knows although it is important not to quote). . .

(Finally, I come to the third major milestone in the work of Mebkhout. Technically speaking, we can say p. 1028

it consists of three (or two) distinct theorems, but so closely linked in the mind of Mebkhout they appear as inseparable. Since January 1978, he proved the appearance "3  $\infty$ -modules": that the restriction  $m \infty$  (where "functor Mebkhout") of the functor "De Rham complex associated" with complexes 3  $\infty$ -modules holonomic is an equivalence of categories (with complex beam C-vector to building cohomology). Knowing already that this functor commutes with functors dualisants it is natural to reformulate this theorem by passing contravariant functor associated  $\Delta \infty$  given by  $C \mapsto \text{Rhom}_3(C, O_X)$

(3)

or friends, often I feel that this dust then accumulated in thick, dense layers, and it formed as a sealed armor, impenetrable, which concerns me through them. . .

723 (\*) This is the duality became the meantime, the general consensus of my students and old friends, "Verdier duality" (both the complex analytic case  $qu@talé$ ). . . (On this subject, for example, the note "The good reference" n ° 82.)

724 (\*\*) It was under this name that the result contained in Chapter III of the thesis Mebkhout. It said he was inspired, for that name (like that of "bidualité theorem") of terminology that I introduced - yet, for me "Local duality theorem" was just another name for the "bidualité theorem" I had cleared, he is importantly, the "geometric" appearance.

This result compatibility (explains Mebkhout) was an important step in its demonstration of what he calls, in this same chapter, "bidualité theorem". (See, regarding the latter, the previous note "The five pictures", part (b).) Demonstration issue aside and the point of view of a "philosophy" or "yoga" was something "obvious" Although the functor of God had to switch to dualisants functors (since there is a good God!). Comically, Kashiwara (Mebkhout who had the opportunity to talk in person in January 1978) does **not believe** that this theorem is true! That is to say how much he was out of his mind, while the geometric vision (style "six operations") lacked. This does not prevent him later after Mebkhout communicates Chapter III (February 1978), to appropriate this result (of course without mentioning its author) in his big article with Kawai already quoted (see note b. p. (\*) page 1005) (prop. 1.4.6 s. 4 loc. Cit.). This is the work which is also suitable without further ado (as the "Theorem reconstruction") the "bidualité theorem" (loc. Cit. 1.4-9 s. 4). This means how much emulated across the Pacific the great masters of the "new-style" born in Paris (in lieu of a "school of Grothendieck" who had vanished without trace. . . ), Are not outdone by their French colleagues.

My bidualité theorem (for discrete coefficients) is also in the same inexhaustible by 4 of the same work Kashiwara-Kawai (Prop. 1.4.2) But then we plunders shamelessly and without thinking twice about the posthumous pupil and unknown,

notoriously left behind by the bosses, we did it rigorously hat to the illustrious colleague opposite, citing as it should be "good reference" provided by Verdier (itself a plundering never named the deceased...).

These deceptions are also notorious among knowledgeable people, and Mebkhout had several echoes in this direction. But obviously, they are considered séantes and welcome for the occasion when it comes to removing the ancestor incitable and unfortunate successor.

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and it is the same to say that this is a functor (anti) equity. This theorem can specify then

by the beautiful **inversion formula** (or "reconstitution" or "bidualité") of Mebkhout giving expression almost as inverse functor

$$F \mapsto \rightarrow \text{Rhom}_{C \times} (F, O_X)$$

(4)

In the process, Mebkhout also shows a **reciprocal** of constructability Kashiwara theorem, know this: if a complex of  $3^\infty$ -modules (or 3-Modules) for coherent cohomology is such that the complex of De Rham complex associated (as a C-beam vector complex) is constructible cohomology then it is holonomic (**Holonomy Cohomological criterion**). In the case of complex  $3^\infty$ -modules, where it poses no regularity question, this implies that in (derived category (where p. 1029 person for a long time not worked, in 1978 and until 1981. . .), The complex (or rather its dual) is "reconstituted" single isomorphism, by the inversion formula.

As I explained elsewhere <sup>725</sup> (\*), from that moment Mebkhout has in hand what it takes to prove Theorem of God also for 3-modules: the fact that the functor m functor restriction

De Rham complex of regular holonomic 3-Modules, is an equivalence of categories. The result inspires less because there are not, apparently, of inversion formula to the key <sup>726</sup> (\*\*). All ways, even his beautiful inversion formula is neither hot nor cold person - starting with its thesis quasidirecteur Verdier (who nevertheless make him the honor to serve as president of the jury). It is not exactly an encouraging atmosphere to remake the technical effort to prove one thing it sure feels anyway, and he feels he has what it takes to prove it. He did attention will once started the "rush" triggered by the demonstration of the conjecture deemed unaffordable (not the Weil this time, but the Kazhdan-Lusztig).

It was, as if on purpose, just the other side whose sudden people were in urgent need. Of Anyway, "everyone" is in such a hurry then use the new "Iron fracture" brand new, which had to appear on the market, and it is so much understood by all that is important not to raise the issue a demonstration - the time it would appear that the work would be done by a incitable - nobody apparently he had the idea, except the person himself, to copy and put the pieces of the  $3^\infty$ -theory already written to prove the theorem it takes 3-theory. It seems that the only demonstration published to date <sup>727</sup> (\*\*\*) is indeed that of Mebkhout published last year (and received in June 1981, the same month Pervert memorable Symposium. . .).

I explained in the previous note (part (b)) a simple principle, inspired by the approach to Deligne coefficients Rham, to retrieve a "inversion formula" (or "bidualité", to use expression (of Mebkhout) within the three-Modules (regular holonomic). I do not know, since we p. 1030 made seminars around the world on the new "cream pie" 3-Modules, whether this very natural approach was clear - Mebkhout has not been informed in all cases. What is certain is if Deligne had reflexes that "my time" was regarded as self-evident, is himself

and from having read the great ideas of an unknown in June 1979, which encouraged him to write also demonstrating the 3-side modules (nearest the algebraic) of its crucial result and it have suggested that "pro" version, actually quite obvious, his beautiful inversion formula. Also, from that moment to Deligne who had paid to know, it was obvious that the ideas were Mebkhout

<sup>725</sup> (\*) See Note b. p. (\*) P. 952 to the note "This work..." (N ° 171 (ii))

<sup>726</sup> (\*\*) We have already seen that there still has - and I come back to this point a little lower

<sup>727</sup> (\*\*\*) Reference: Another equivalence of categories, Compositio Mathematicae 51 (1984), 63-88.

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give the coefficients De Rham missing, at least in algebraic geometry on a body characteristic zero; the obvious thing was to encourage him to make the adjustments that were necessary to state a theorem of God (or rather, of Mebkhout in this case) for algebraic varieties complex <sup>728</sup> (\*).

But other times, other customs. It will not be said that a new start in the cohomology of varieties Algebraic was accomplished by lonely and persistent efforts of an unknown wave, claiming a deceased which nobody in the beautiful world for ages, not dares to pronounce the name <sup>729</sup> (\*\*). He will not be said that the revival will come through the kind of mathematics, specifically, that for ten years the heirs of deceased were buried, while sharing the oripaux. Mebkhout innocent, if he wanted to "survive" (and "pierce" p. 1031

had only to follow the path mapped out the "new style" <sup>730</sup> (\*), like other bright young people (and even young) were quick to do so. What mania also cite the source (unspeakable) of his ideas, when it is so easy to drown a fish and mention only those who **should** be cited. mebkhout I believe that your account is good!

You have landed in a world for which you did not - and I am however happy for you, that you am not for **this world - there**. You have done the work you felt you had to do without worrying about you to mode, without making calculations of returns, simply trusting your own instincts - even if it means your path in the desert. You have done **your** work, rather than watching for subtle signs (and less discreet) of those

who decide what is good and decent and what is not. You did not tacked to please, you do not have says "white" when you see black or vice versa - and it is with **your** eyes as you look. I do not have to thee congratulations - you did not seek the praise nor mine nor those of anyone. And all this, I happy for you and for all.

### **b3. The master role (2) - or the gravediggers**

**Rating** 171 (xi) (May 5) <sup>731</sup> (\*\*). The natural question here, of course, is whether there is an algebraic geometry formalism "six operations" for 3-modules (or "crystal") not necessarily like DRM, which "Coifferait" those I had introduced into coherent and discrete case - assuming first, to secure the ideas, which is on the body C. The first difficulty is that the concept of 3-consistency is not stable by natural concept of tensor product crystals by either the inverse image operation analogue <sup>732</sup> (\*\*\*). To hope for a formalism of the six operations must (therefore work with a category p. 1032

<sup>728</sup> (\*) As I have already had occasion to point out, in the algebraic framework, when held to paraphrase the discrete coefficients algébriquement constructible, necessary to impose complex planned 3-modules, in addition to the condition Holonomy and local regularity, regularity condition "on Deligne - Mebkhout" to infinity.

<sup>729</sup> (\*\*) We have not yet found a way, indeed, to find alternative references for EGA and SGA. But these Angel markings contain no allusion to a name that you must stay. As everyone knows, the SGA means an acronym algebraic geometry seminar hosted by the care of Mary Wood, and under the leadership of a number of mathematicians all what is good and perfectly nameable like M. Artin, JL Verdier, P. Deligne Illusie L., P. Berthelot, N. Katz P. Jouanolou or other less known but equally quotable. Obviously there was then a flourishing school of geometry algebraic, called "Marie du Bois", the heart and soul was the brightest among the names cited. For more details about this " **school of Mary Wood** " and the acronym AMS is the expression, see in particular the notes "Eviction (2)" and "funeral -" Im Dienste der Wissenschaft "" (n ° s 169 1 and 175). (See also p. 899, paragraph 3, the note "The double meaning - or the art of the scam ", n ° 169 7 ).

<sup>730</sup> (\*) See, on this style (which took the place of a "school of Grothendieck" disappeared without a trace...), The end of the note "Congratulations - or the new style", n ° 169 9 .

<sup>731</sup> (\*\*) This sub-rating comes from a footnote on page note "This work..." (N ° 171 (ii)). See the reference to this sub-rating, placed toward the end of the note cited (p. 956).

<sup>732</sup> (\*\*\*) (22 May) Mebkhout informed me that he has proved that the condition holonomy and regularity is stable operations tensor of total product ( $O \times$ ) and the concept of mirror image, and that the functor of God **contravariant**  $\delta$  commute there. 792

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bigger even than Cree

\*

coh (X), perhaps that of the crystals "quasi-coherent" (in an obvious sense) - but suddenly there is little hope of recovering a bidualité theorem! In addition, the former natural functor  $O$  voltage scalar  $x \rightarrow 3 \times$  obviously not switch to the tensor product - thus, even there would be a theory of six operations for the crystals, which would extend the (morally known soon now through Mebkhout) crystals Rham - Mebkhout (obtained by "transport structure" in From the "discreet" theory, via the functors of God), it does not extend that of  $O \times$ -modules consistent <sup>733</sup> (\*). This may not be excluded, however, there may be a "global duality theorem" Version quasi-coherent crystals, for a proper map (say) type schemes ended on a body characteristic zero, which "cover" (in an obvious way) the duality theorem "known" (morally, by transport of structure yet) for crystals Rham - Mebkhout and the theorem of similar duality known (without quotes) in the coherent case <sup>734</sup> (\*\*).

(I was quite flabbergasted that Mebkhout itself is either not asked at least this last question, from p. 1033 the moment when he had arrived at the formulation of the duality theorem "absolute" (corresponding to case the purpose variety would be reduced to a point) - recently even he did not seem so much "feel" <sup>735</sup> (\*\*\*)).

This makes typing for me how a certain "philosophy", which in the first half of the sixty had become second nature to me, and (it seemed to me ...) to my students too - how this philosophy has been forgotten by all, starting with those who are responsible to make gravediggers, rather than transmitted. And I see that it is there as the main cause of this staggering stagnation experienced by after I left a theory (that of the diagrams cohomology) that I left booming.

Admittedly Mebkhout placed himself in the transcendental analytic complex context, instead of the context schematic. This introduced considerable technical difficulties, somehow "parasites" when it is to reach an understanding of the essential variance phenomena. Again, his elders failed in their task, which was to put their experience, gained at my touch, available to the new (For cons, the functor of God covariate m no switches, and it transforms ordinary mirror image in mirror image Extraordinary.) It can be shown, using this result, there is no formalism of the six operations for the coefficients De Rham - Mebkhout that "extends" the two fundamental operations already known tensor product and inverse image.

In particular, the DRM category  $\mathfrak{b}(X)$  has no operation "internal Hom" (playing the role of Rohm), and  $f: X \rightarrow Y$ , the functor  $f^*$  does not admit in general assistant right  $f^*$ . The functor  $f_!$  introduced now Mebkhout by  $(X, Y$

smooth and clean  $f$ ) is an assistant to **the left** of  $f^*$ . (NB The  $Rf_!$  operation  $!$  On the coefficients De Rham - Mebkhout was defined such that the functor of God **covariate** are switches, and similarly for  $f^*$  - wrongly or rightly. . . )

This shows that in terms of "natural" operations that are available in the context Rham -Mebkhout, these do form **not** as such a "theory of the six operations", but a sort of dual theory. The question therefore is to see how this one extends to 3-Modules (quasi-coherent say) that are no longer supposed holonomic and regular (e.g., holonomic without further - provided that is retained by the tensor product and inverse image). he would seem in particular that the global duality formula can be written for complex 3-cohomology modules consistent (or nearly consistent only), and a map  $f: X \rightarrow Y$  any separate schemes of finite type on a body  $K$  because. zero (say), so as to style both theorem consistent duality, and the discrete duality, at least as follows: the Dualising functor "exchange" functors  $f^*$  and  $f_!$ .

733 (\*) It is appropriate to reformulate this assertion rather in terms of a "dual theory to six operations", see note b. p. former.

734 (\*\*\*) may be considered such a duality theorem in three different forms. Is saying that the functors in dualisants up and down "exchange" functors  $Rf_!$  And  $R^* f_!$ , is saying that two properly defined functors  $Rf_!$  and  $R^* f_!$  are associated with one another, either by writing a "projection formula" (which cover one and the other statement):  
 $R^* f_! (R^* f_! (F), R^* f_! (G)) \approx R^* f_! (f_! (F), G)$

735 (\*\*\*) (June 8) Mebkhout yet assures me that he had indeed asked for a long time. If I had the impression of Rather, it is surely that this issue remained to him entirely platonic.  
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come (as I had made them...), and thus guide (or at least the light) in his choice of investments, particularly.

Inform and guide but is also **used**, although they had long been without opted for sharing the role of the master.

#### **b4. The dead pages**

**Rating** 171 (xii) (May 5) <sup>736</sup> (\*) Mebkhout told me that before I talk to him when we met there two years <sup>737</sup> (\*\*), he had never heard even utter the words "six operations" - although he wondered of what "operations" I wanted to talk! Obviously he had never thought (nor anyone else, it seems, except me) to review the main ingredients of a certain formalism cohomological simple, note that there were six or functors bifunctors fundamentals, grouped into three pairs functor assistants, with such arrows and compatibilities etc. These were things that seemed so obvious, that I imagined that any player (either "Residues and Duality" outlining the elements of p. 1034

consistent duality, or SGA SGA 4 or 5 exposing the discrete elements of duality, with essentially the same form elsewhere, will be amused (as I had done since the fifties without going to After I recognize. . . ) To set up for his own use a form more or less systematic and or less complete, the main isomorphisms and major compatibility - for that is just, and in no other way, we manage to penetrate the mind of a new language, to assimilate closely at the to "own". This is so and not otherwise, surely, had done the pioneers of calculus for achieve a delicate and sure intuition of the infinitely small at a time yet where the conceptual tools lacked to apprehend as rigorous guns appeared (or reappeared) later. . .

With a decline of twenty years, I realize that in the "reference documents" cited, made with the most great care, even brilliantly - whereas all the "real work" (according to the current desiderata) is made, culminating in "the" main duality formula, the formula of addition between  $Rf_!$  And  $R^* f_!$  (practically the only judged worthy of attention and effort, even forget the next day, as we forget the trees when we did not see the forest. . . ) - as yet in all these texts the **main** is not said and has not gone from author to reader (Assuming it is seen and felt by the author himself). "The main" is a "yoga", a "philosophy" a conductive wire foolproof through (in this case) the cohomology jungle Algebraic Geometry (and elsewhere). We can develop it extensively over fifty pages or a hundred, when "everything is fact "(so called), as can also be content to evoke in a few pages and leave to the reader care to develop for its own guidance as far as it deems necessary for its own needs, or his own satisfaction.

It is these few pages then, whether on the "six operations", or on the grounds, or on many others Things <sup>738</sup> (\*), pages that I felt strongly but why I was not able to feel how it was important that I write them - they are the ones who missed, especially in my written work. I was absorbed by meticulous and endless tasks at (service of all, the big "work room", the only one that was p. 1035

supposed to publish - I was not able to feel that there was more essential pages, I was **the only one** in power

to write. **Much** as I had to say has not happened in the pages written, but word of mouth only  
736 (\*) (May 22) This sub-rating, as the previous one, comes from a note b. p. on page "The work of..." (n ° 171 (ii)).  
See the sign referring to the end of this note, p. 957.  
737 (\*\*) It is about the meeting in the note "Dating from the grave" (n ° 78).  
738 (\*) After these lines were written, I could see that for les six operations, I@ here error - in fact, I  
was misled by publishing-killing SGA 5, which Illusie took care to eradicate any trace of "yoga for six operations"  
I had developed extensively in oral seminar, with full form copiously commented.  
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- when it wanted to spend! Or, in a pinch, it was in between the lines, perhaps, endless  
volume basis - but there he is someone today who can read between the lines?

The key, then, is what was given daily to those who, in my mathematician@life were  
figure "close", and first of all, to my students. That was one thing that was obvious, nothing deliberate.  
The idea would never have occurred to me that somehow I was investing the well of a **power** considerable.  
It@not that I felt the strength of that I designed and transmettais but that strength, too,  
was obvious. In my opinion, surely, mathematics, at least, "strength" and "beauty" were and remain a  
same thing. The idea would not come to me that we can abuse it, these things filled me  
peaceful and intense life, have to live and create. When I left, so my faith can not  
most unexpected, I had about them a shadow of concern. These pages that I had never thought of writing - it  
There was no doubt in me that their message had long welcomed and registered, and that these "close"  
would be so many living pages, that would tell the message and enrich what they would better  
to make.

Those to whom I addressed with confidence and respect, as to younger brothers and I in  
recognized myself, chose to bury and keep quiet. And when he came, true to himself, in whom they  
recognized me, they filled all chose to leave their front doors closed - a stranger and  
intruder. I do not know you ! And those unwritten pages, those pages say in vain, become dead in these pages  
opulent houses with haughty and closed doors, we had somehow found the challenged brother in the  
himself, in long and groping labors. Only, he had to fight his way through the tangled jungle  
the thousand and one hundred thousand volumes. One who has been there, even if he was lucky, like me once, of  
have the fraternal aid of experienced and caring guides, knows what I mean. . .

He has made a way, painfully, as the days and years - a chugging way without a compass  
He sometimes seemed to me afterwards, no other compass or, at least, a flair that is still trying to  
through a (painfully experience and hard-won. He did not rewrite his use these ready-made pages, p. 1036  
compasses those pages, pages become haughty died in homes - if only in bits scattered. he  
wrote **other** pages, **its pages** , painfully hers. He wrote them chugging along stubbornly in  
the indifference of all. Yet these pages often clumsy and worthy of a cad, my brilliant and  
plush students once (if they had bothered to read them) would certainly regarded with pity and  
nothing to see - these are pages that **were to** be written, as a natural consequence, "obvious", these  
pages I had never even thought to write, so it seemed to me self-evident. . .

#### c1. Hatching a vision or intruders

**Rating** 171 1

(April 15) 739 (\*) Building on the recent visit home from my co-buried Zoghman Meb-  
khout in person, I would like to give some hot details on all its strange mishaps, such that  
made me a hand himself in snatches sparse here and there during our conversations.

Zoghman had the honor of an "interview" with his "boss" 740 (\*\*) JL Verdier on three occasions. The

739 (\*) (30 May) The three following notes (n ° s 171 1 to 171 3 ) were written between 15 and 18 April (1985), at a time when  
"The Apotheosis" is still reduced to a rating of ten pages. These have greatly expanded over  
in May, following the relaunch of the reflection on the Four Operations, triggered by the passage of home Zoghman  
Mebkhout. The ten pages have become more than a hundred, of which almost all is a later vintage than the three notes  
that follow. It follows some partial rehearsals, certain facts or events being mentioned or described under  
different light, in earlier grades and in those that follow. For the sake of preserving the spontaneity of the writing,  
I did not want to make adjustments to eliminate these repetitions.

740 (\*\*) (24 May) Mebkhout insists that the term "boss" (even with quotes) is moved here. Since its inception in 1972 until  
795

First up was in 1975 - he needed a technical result, which was contained (as it appeared  
thereafter) in bidualité theorem for discrete coefficients analytically Building - a  
when Zoghman even know the concept of constructability. (This is a concept that I had introduced  
since the fifties, and had been taken as part of the topology spreads in SGA 4.) At this point

this notion was not "well known" in the analysis, as it (is today. It is that it is

p. 1037

the notion exactly he needed for his work. Houzel (which followed SGA 5 together Verdier, but that was to have almost forgotten what I had told), advised him to go to Verdier. It was where the first "interview" with the great man. Verdier taught him when he asked (two discrete complex that had "duals" isomorphic were isomorphic) was true under certain conditions Technical (the "constructability", actually), he would find exposed in the manuscript that he would give him. It was one of the "good reference" <sup>741</sup> (\*), where (among other feats of the same ilk) he pretends to invent buildable beams and discover the bidualité theorem (and its proof), things he had learned through my mouth twelve years earlier (1963) <sup>742</sup> (\*\*). He breathes not a word of me about it, not more in this interview that the manuscript was published the following year. Zoghman anyway go again filled and full of gratitude for the great man, who provided him with exactly what he needed at that time, and in the following years again, where the concept of constructability would play a crucial role in all its work.

It was in early 1976 that it began to be interested in the duality, and to be intrigued by the analogy of formalismes duality that I had developed in the consistent case and the discrete case "spreads", which had been taken over by Verdier in the topological discrete case. This is at a time when, for years, this formalism fell obsolete, and where my students had instituted a boycott (tacit and rigorous on derived categories, p. 1038

constitute natural language. The notion and the very word "formalism six operations", which was One of my main key ideas from the fifties and throughout the sixties, was de-came (and remained until today) strictly taboo soon after my departure. (When Zoghman came to me two years ago <sup>743</sup> (\*), he had not heard even pronounce the word "six operations", and know first what "operations" I meant - as I thought it was for twenty years a concept familiar to all!) This means that the conditions were adverse to move in this direction, where he was sentenced to work in complete solitude. This does not prevent him from 1976 to release a duality theorem on the non-singular complex varieties, which "cover" both the duality theorem Today, he did his job without a boss, in making do by yourself. Verdier was merely president his thesis committee. Apart from that its role was limited to communicate Mebkhout "good reference", which has been very useful, a

when SGA 5 still continued to be kidnapped by the combined care of my cohomologistes students (and for purposes precisely operations such as that of the "good reference." . . ).

<sup>741</sup> (\*) This section JL Verdier, homology class associated with a ring, Asterisk n ° 36 (SMF), p. 101-151 (1976). It is issue in detail in the two consecutive notes "The good reference" and "The joke - or" complex weight "" (n ° s 82, 83), and more briefly, in the note "Episodes of escalation" (n ° 169 (iii)), with episode 3.

<sup>742</sup> (\*\*) From the second half of the fifties I had been interested in notions of "constructability" of all kinds for discrete beams (in the algebraic sense, complex analytical, analytical real, piecewise linear - meanwhile the context of moderate topology. . . ), In addition to the concepts of consistency, as natural concepts to express conditions finiteness sheaf in the frame, and had raised the issue of the stability of these concepts by the "six operations". It©further development (in 1963 and following years) of étale, who brought me back on these issues in spreads, and develop techniques (unscrewing and resolution) used to treat by a standard method, which apply also to transcending context of complex analytical and algebraic varieties complex. The bidualité theorem, valid (and with the same demonstration) as part spreads (for purity and resolution) and the transcendent context was cleared by me since 1963. It appears also in the first set SGA 5 (in 1965), where he survived the massacre of publishing-Illusie 1977.

<sup>743</sup> (\*) It is about the visit in the note "Dating from the grave", n ° 78. For comments on the boycott instituted the "six operations", see also note "The dead pages", n ° 171 (xii).

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Serre and discrete duality (which he calls "Poincaré duality-Verdier") in terms of a duality statement Modules for the 3-complex (which also contains a comprehensive statement of duality for complex opera-Differential tors). The "coefficient" it takes are also a generality that far exceeded the If Serre (limited to locally free beams) and Poincare (limited to discrete beams locally constant) faithful that in mind that I had introduced into these themes with the formalism then generally repudiated the "six operations".

When Zoghman explained this theorem it two years ago, I felt both its interest, which was obvious for me, and its limitation, because in the minds of "six operations" it was also clear to me that " good "statement was to be a statement about a morphism of analytic spaces  $f: X \rightarrow Y$  as (for example) a statement adding between two RF functions  $\iota$  and  $Rf \iota$  . It is true that the act of placing in a transcendent environment introduces significant additional difficulties, which strongly acted (I think) to obscure Mebkhout for simplicity essential algebraic mechanisms in



duality - while nobody around him, and especially not among those who were my students would have known (or deigned. . . ) Make him feel. Still, he had pinpointed a "principle" significant - the one that Theory 3-modules (I myself prefer to call "crystalline modules" <sup>744 (\*\*)</sup>) provides a "denominator common "to" cap "the (phenomena (of duality, in particular) discrete cohomology and cohomology p. 1039 consistent. The momentum then, encouraged by someone who was "in the know" and with a minimum mathematical instinct of <sup>745 (\*)</sup> and of benevolence, no doubt he would have developed in the space of three or four years following a full formalism of the six operations in the context of algebraic geometry characteristic zero (at least), providing a "paradigm" purely algebraic faithful of the same formalism (repudiated, it is true) in the transcendent framework for C-vector bundles algebraically buildable.

Feeling that he had discovered something important, Zoghman all happy seeks and obtains an interview benefactor to expose his result. It was **the** answer, precisely, the question I asked Verdier ten or twelve years ago, without having the air securely holds <sup>746 (\*\*)</sup> - there are chance he had even completely forgotten. Anyway, his benevolence towards the young man who came from nowhere and did things on which he, Verdier had drawn a great feature long ago, was exhausted. He did not even want to hear the explanation of the ins Zoghman and outs and the proof of the theorem. He made it clear in substance (and politely) that he, Verdier, no longer believed in Santa and the young man had better pack up.

Extraordinary thing **anyone** around Zoghman "not hang" this result <sup>747 (\*\*\*)</sup> - probably it was <sup>744 (\*\*)</sup> For the reason (obvious) this terminology "crystalline", reflecting a more intrinsic understanding of 3-Modules (my students had learned by me and they are long forgotten), see the comments in the note "My orphans" (n ° 46) (in particular p. 179) and the sub-grade n ° 46<sub>4</sub> (p. 188) (x). About "blocking healthy schools" against links evident from the philosophy of yoga Mebkhout with the lens that I had emerged in the late sixties, see note "Spoofing" (n ° 85 @pp. 350-351).

(x) (May 24) See also note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)).

<sup>745 (\*)</sup> It is not my cohomologists former students are devoid of a "minimal mathematical instinct" - otherwise no of them could have done with me the good work he has done. But this instinct is misguided or blocked syndrome Burial of the master.

<sup>746 (\*\*)</sup> (June 5) On this subject the note "ancestor" (n ° 171 (i)), including the note b. p. (\*) On page 946.

<sup>747 (\*\*\*)</sup> (June 3) There has been a misunderstanding. As was said in the note "Three milestones - or innocence" (n ° 171 (x), page 1026) this theorem had the gift to amaze often casual contact. But it would appear to be far remained platonic - the theorem does not become a tool, something we know and which are used without thinking. this is surely related to the fact that not one who looked forward to the obvious beauty of the result was one of those who "set the tone"

and decide what is "important" and what is the "bombinage". (And it is not unusual, for these times, the "Bombinage" of yesterday becomes the "cream pie" of today. . . ) - In his comments of 22 April, Zoghman writes: "... There was an embarrassment in front of this theorem. Some secretly envied. But very few people who encouraged, 797

too "grothendieckerie" years (sixty, we exceeded that these days, thankfully! Maybe I p. 1040

was it two years ago, the first person he met, who feel the importance of the result and "Philosophy" news that he carries the seeds - that of a vast synthesis between "discrete" aspects and "Differential" aspects (or "analytical") in the cohomology of varieties of all kinds (and algebraic Analytical to start). This theorem, which is one of the chapters of his thesis, was finally published in *Mathematica Scandinavica* in 1982 (vol. 50, pp. 25-43). The same article was submitted to the *Annals of Mathematics*, who made it clear to young presumptuous it was not necessary to be published in the periodical of standing.

Even still today, this theorem is usually ignored or scorned in the beautiful world, while has already embedded this new philosophy, via the theorem of God (aka Mebkhout) gave means a dramatic renewal in the cohomology of algebraic varieties. But "all the world, "including my cohomologists former students (one day yet I have known endowed with a healthy instinct mathematics), rushed en masse to the new "cream pie", namely a powerful tool (that "Everyone" yet affects name only by allusion or circumlocution as "the relationship between Plot beams and holonomic differential systems ", or as" that which should have normally find place in these notes " <sup>748 (\*)</sup>...), and the " art "(the intersection cohomology), while the **vision** p. 1041

Innovative who has identified the tool is ignored as much as before, and the father of one and the other is treated stooge.

The situation here is the same as my vast unifying vision topos, derived categories, six operations, cohomological coefficients and beyond again, that memorandum. It is this vision that came out tools like the étale and crystalline cohomology, that this same "everyone"

used today as we turn a crank, while the vision itself, powerfully alive yet the day I left, was buried the same day. And I see clearly that the staggering I see stagnation in splendid about 749 (\*) Fifteen years after leaving booming, is not due to a lack of intellectual capacity or donations (which are brightest in more than one of those I so well and not well known) but the provisions of gravedigger, or shameless nepotism or both - provisions the opposite of innocence which recognize, and which find it simple and essential.

To develop its new philosophy Mebkhout was inspired by the spirit of derived categories and six operations, at a time when the derived categories were treated humbug grothendieckienne and where he had not had the opportunity to hear even pronounce the name "six operations". Today, with the rush on quite the contrary. "

748 (\*) This is a quote (from memory) the "memorable article" Beilinson-Bernstein-Deligne (written by Deligne) he was issue in the note "The day of glory" (n ° 171 (iv)). For details on this paraphrase one, worthy to be remembered (As a reminder and as a warning...), And the ins and outs of the context, see note "The prestidigit-feeler "(n ° 75)". The quote that preceded ( "Plot of the relationship between beams and holonomic differential systems") is extracted from the article Beilinson- Bernstein (the year 1981) which will be discussed in the following sub-rating ( "The mafia ", n ° 171 2 ), where we will have the advantage of also getting to know the contribution of Brylinski-Kashiwara at flowering

This kind of style in the service of the same scam.

749 (\*) I speak for the first time this impression of "sullen stagnation" at the end of the note "Denial of inheritance - or price of a contradiction "(following" My orphans ") n ° 47 (p. 195). This impression has only been confirmed in the year that has passed since the writing of this note, with the same restriction, essentially, as I express in the sub-grade n ° 47 3 in note cited: the work of Deligne on Weil conjectures (Weil I and II) and the new departure that followed the "rush" on the theorem of God (eliminating and God, and his servant Zoghman), and the intersection cohomology. But these successes localized strike me as disproportionate to the brilliant ways, even exceptional ones I know of for having "settled" for this "splendid subject" - while fifteen years have since my departure; and also without common measure with the richness and vigor of key ideas that I had inherited, and I found today bloodless. . .

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the new tool appeared inseparable derived categories, were exhumed in great fanfare in recent in concealing the name of both the one who had emerged from nothing for years of solitary work, that of one who was inspired, too lonely, to finally hatch a new theory coefficients connecting topology, complex analysis and algebraic geometry.

The Deligne, Verdier and others flock to the brand new innovations shouting (with discretion p. 1042 rigor and genuine, it goes without saying) "it@me, it@me!". None of them managed to find itself courage and loyalty to itself, to mature a vision in solitude, wear heavily for months and for years, far applause, as they would only see and they could share what they see with anyone in the world.

But I digress, it@time I return to my story of **the birth of a vision** . It is from the same

1976 Mebkhout which demonstrates the duality theorem that "cap" Poincaré duality and duality of Serre, he comes to the idea of the equivalence of three categories, which respectively embody the aspect of "topological "appearance" algebraic "and appearance" analytical "(transcendent) of the same reality, of the same type objects. In the context of a general theory of "cohomological coefficients" 750 (\*), I will call these objects "coefficients Rham - Mebkhout" 751 (\*). If X is a smooth analytic space 752 (\*\*), there are on the one hand p.

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750 (\*) This idea of the " **types of coefficients** " different, each of which was presented to me as a particular incarnation of formation six operations ism (and bidualité), identifying more or less near the "type coefficients" the finest of all, the type "absolute" or "universal" or " **pattern** " - this idea was perhaps the main idea force that guided me throughout the years sixty, and especially since 1963, in the development of my vision cohomological algebraic and other varieties. The strength of this idea in me is clearly visible from the very first note I devoted to a retrospective of my work, and these vicissitudes the hands of fashion: "Orphans" (n ° 46). I return insistently in various places of reflection on the Burial, especially in "The melody in the tomb - or sufficiency" and "The tour of building sites - or tools and vision (n ° s 167, 178). It is also the first mathematical topic among those buried under the care of my former students cohomologistes and those of fashion, I think develop as a result of Crops and Seeds to give him instead it deserves in my mathematical thinking.

Strangely, this central thrust of my work cohomological, and algebraic-categorical structure (easy to background) that expresses it, was never explained in the literature, not even by me during the sixties (x). She appears between the lines in my written work, and was particularly conveyed in oral communication. In my mind, it was obvious that one of my students do not fail to spend a few days or weeks it took to present systematic form this set of ideas, whereas myself was fully occupied with the tasks of the foundation

EGA and SGA.

Looking back, I realize how better account of non-formal texts (if only a few pages in this instance  
ence, and without any effort to accurate and systematic formulations), making just feel these "key ideas" rarely  
appointees who are hidden behind texts often technical appearance - how such texts are important  
to guide researchers, and bring occasionally a breath of air in a literature that tends to stifle by  
its technicality. In this regard, Zoghman told me also that some passages of this kind that he found in the texts of  
my pen has been of great help. Among them, it still me recently highlighted the few words of introduction  
tion that I had joined the volume of Hartshorne "resigned and duality (volume essentially exposing the formalism of six  
operations that I had developed in the second half of the fifties, in the coherent framework). I now measure  
how that introduction would it even more useful if I had bothered to include, not least a page or  
two non-formal, explaining the "yoga of six operations" and stressing its importance as a pervasive thread  
in building cohomology theories still waiting to be born. . .

(X) (May 24 and June 1) After these lines were written, it appeared that from the very beginning of the oral seminar SGA 5 (in  
My second presentation), I had taken great care to develop extensively the form "abstract" of the six operations,  
would dominate the entire upcoming seminar. (On this subject see the note b. P. (\*) Of 8 May in note "The Ancestor" n ° 171  
(i)

page 942.) In addition, throughout the oral seminar, I did not fail to constantly refer to the ubiquity of the formalism  
cohomological that I developed, in principle, to all sorts of other types of "coefficients" as "coefficients  
l-adic ". Illusie was careful to root publishing killing both the Narrative dedicated to the formalism of the six  
operations, that any reference to a vision of "cohomological factors" beyond the particular context being  
Senior Seminar.

See also in this regard the note "The dead pages" (n ° 171 (xii)), and also "The unnecessary details" (n ° 171 (v)), part b  
( "Machines to do anything...").

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category ( "derivative") of the C-vector complexes "Building" X, Cons

\* (X, C) or simply

Cons \* (X) ( **aspect of "topological "** ), the complexes to cohomology coherent beams 753 (\*\*\*)  
generalizing complex infinite order differential operators, I note DRM

\*

∞ (X) ( **appearance "analytical**

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tick " transcendent), and finally the complex category 3 ∞

X -modules beam cohomology co-

herent, generalizing complex ordinary differential operators (of finite order), I note that DRM

\* (X)

( **Look "algebraic"**  ). There is a functor scalar expansion tautology of coherent Ring 3 x  
to the ring 3 ∞

X

i: DRM \* (X) → DRM \*

∞ (X)

inserting into a functor diagram (essentially commutative):

DRM \* (X)

i

//

m

©

OOOOOOOOOO

DRM \*

∞ (X)

m ∞

wwoooooo

(1)

Cons \* (X)

where the oblique arrows are arrows "associated Rham complex" 754 (\*), which is none other than Rhom D (Sp \*, .),  
where D = D x or D ∞

X, Sp and where \* is the "resolution Spencer" W X by locally free 3-Modules  
754 (\*).

The existence of vertical arrows comes from the "constructability Kashiwara theorem," which implies  
the Rham complex associated with a 3-holonomic modules of complex beam cohomology  
Plot analytically. Kashiwara had demonstrated this important theorem in 1975 755 (\*\*), in a

Optical completely different though. He worked with a single 3-module holonomic, which he took the De Rham complex and proved that its cohomology is constructible. Until September 1979 and the "Rush" further triggered by the theorem of God, him nor anyone else in the beautiful world was working in the minds of derived categories, and the idea to write the vertical arrows in (1) was occurred to anyone!

Once the three arrows (1) written as arrows between derived categories <sup>756 (\*\*\*)</sup>, the question arises if those are equivalence classes. Mebkhout was convinced in 1976. The conviction was him endrassant came an array of ten typical examples (reproduced in his article with expository p. 1045

Le Dung Trang <sup>757 (\*)</sup>) of building C-vector bundles can be called "elementary", which <sup>751 (\*)</sup> (30 May) In the note (written later) "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), I am terminology a little different, denoting by "coefficients Rham" (short) the "same object", which we will give three here **descriptions** (or three "photos") **different**. Two of them have the "coefficient de Rham - Mebkhout" (or simply, "to Mebkhout"), "infinite order" and "finite order" respectively.

<sup>752 (\*\*)</sup> (30 May) In the original version of these notes, leaving me away by my predilection for the view "geometry algebraic "I had assumed that X is a variety **algebraic** C. This does not fit the context in which was placed Mebkhout initially, besides it made me express a variant of the "theorem good God," for complex  $3 \infty$ -modules, which is true as it is assumed that when X clean. So there were misunderstandings in my mind, and Mebkhout had to gently remind me to order. Retyping the net these pages, I made the adjustments that are needed.

<sup>753 (\*\*\*)</sup> As regards the definition and first soritiaux facts concerning the modules and 3-modules theory, the reader Refer to note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), especially the parts (a) and (b) ("The album" De Rham coefficients ", " and "The formula of God").

<sup>754 (\*)</sup> (May 24) See note cited "The five pictures..." (N ° 171 (ix)), part (a).

<sup>755 (\*\*)</sup> Masaki Kashiwara, On the maximally overdetermined System of linear differential equations, I Publ. RIMS, Kyoto university 10 (1975), 563-579.

<sup>756 (\*\*\*)</sup> Strictly speaking, it would probably be more correct to say that these are full subcategories (defined by conditions of "buildability", or consistency, Holonomy and regularity) derived classes in the ordinary sense.

<sup>757 (\*)</sup> Lê Dung Trang Zoghman Mebkhout, Introduction to linear differential Systems, Proc. of Symposia in Pure Mathematics, Flight. 40 (1983), part 2, p.31-63. Zoghman advised me this short article as the best introduction that exists in 800

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are of the type as those who constantly involved in the "unscrewing" bundles familiar with the theory of étale. Of this crucial year 1976 for each of these beams, it happens to build a remarkable holonomic complex, both  $3 \times ($  " **algebra** ") on  $3 \infty$

X ( " **analysis** "), having (the perspective of six operations) an algebraic or analytic cohomological meaning very simple, and whose the De Rham complex is the beam in question. Remarkably, as he left a bundle building and not a beam complex, in a number of cases the holonomic complex gives birth is nevertheless him not reduced to a single cohomology bundle. It showed him well that, in the spirit of "six operations" (which he did not know the name...), if equity ago had, she could not infer an equivalence between modules beams categories (C, or D) themselves, but she took his meaning only passing derived categories.

For me it is clear that **the act of creation**, in this case, was to see and write both  $m$  and  $m$  arrows  $\infty$  **obvious**, and nobody yet had deigned to write - to ask the question "every beast" if it would not, sometimes equivalency classes, so providing an algebraic interpretation differential, and another differential analytical, the beam topological notion (or complex of beams) Building C-vector. There was the **matter, and a clear awareness of the crucial nature this issue**, its scope - and by the same token, and as a matter of course, an interior attitude **assumed** this issue, which would make the bear to completion. The "experiment" with preliminary the "typical" or "elementary" examples was a first step in that direction.

It was there not childish and essential, that which is done only by one who knows how to be alone. Once this not then accomplished, the first of my students came cohomologistes using unscrewing the technical and Resolution learned my contact in SGA SGA 4 and 5, was able to prove it in a few days, or p. 1046 few weeks - as long it catches only course he feels (as Mebkhout had felt and his guts) meaning, the **substance** of the issue. But there was not one of them, not even Deligne who withdrew to clear the unifying vision that goes **beyond** the key idea of "six operations " <sup>758 (\*)</sup>, which was missing to connect discrete continuous coefficients and coefficients - not one who has been able to see the scope, obvious yet Mebkhout of ideas, this unknown wave that stood still Grothendieck©spitting image. . .

As for the "unknown wave", reduced to his own and his reading, the question of equivalence categories lence must seem to him (rightly more) like the obvious thing and most of the child

world, or to come to the conviction that they were definitely there equivalencies. By cons, for lack of experience and encouragement by more experienced older than him, he was a world of the demonstration, which long seemed completely out of reach.

Still, he manages to find a demonstration after a year and a half already, first for the boom  $m^\infty$ , in March 1978. He said that psychologically, my comparison theorem for co-literature to philosophy he has developed since 1976. It also found there, in the bibliography, a list (full?) of Mebkhout publications on this subject, at least until 1983.

758 (\*) (June 5) In my rereading, this formulation seems hasty and somewhat i: next key idea of "reality Actually my." six operations "was inseparable from a" philosophy of the coefficients", which included (and very clear at least since 1966) a "theory of the coefficients De Rham" (closely related to my crystal ideas) with the same formal properties essential that the theory of l-adic coefficients, and forming with them (for variable) as many "achievements" different the same type of ultimate object, the "pattern". The work of Mebkhout completed between 1972 and 1980, appears to me as a first

great step towards achieving this intuition - not for whom everything was ready, almost, at least in 1966 with the start crystalline yoga, when the problem of a theory of the coefficients De Rham was found clearly stated in my mind at least. If this was not done by any of my students cohomologistes and this from the sixties, it seems mainly due to locking mechanisms of spontaneous creativity, which missed by no one. On this subject the note "... And hinders" (n ° 171 (viii)).

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homology De Rham algebraic and transcendent, he was a great help, to put on the road demonstration. For some reason I did not quite understand, he also considers his theorem (ie said that the functor  $m$  "of God," especially for Mebkhout not say ..., is an equivalence) as being a "generalization" of my comparison theorem. From that moment he knows he has the tools p. 1047

need (with the technique of solving Hironaka) to also handle the case of  $m$ , by far the most integrated resant for algebraist surveyor like me. He, as an analyst, was initially attached to the case the functor  $m^\infty$ , which had a preference 759 (\*). He returned to the issue, which seems to show him some accessory after the defense of his thesis and demonstrates the following month (March 1979) that the functor  $m$  (the one everyone today uses circumlocution calls without ever writing it, not having to appoint a nameless author. . .) Is an equivalence of categories 760 (\*\*). So, it follows that the functor "ring of change"  $i$  ranging from "algebraic" (in which he was interested even that far) to "analytical" (transcendent), was also an equivalence.

\*  
\*  
\*

In March 1978 Mebkhout was his third interview with his "benefactor" Verdier, he had not p. 1048

seen for two years. He then explains the ins and outs of the (future) "theorem of God," he modestly calls (evil took him!) "equivalence Riemann-Hilbert". Looking back, Mebkhout was confident that his explanations had to go over the head of Verdier. What is certain is that Verdier went absolutely not aware that his "protected" had to submit ideas that deserved some attention. It speaks to anyone around him, not even Deligne, who teaches the theorem

759 (\*) (May 24) Another reason, stronger perhaps, is that in the case of  $3^\infty$ -modules he had a beautiful formula inversion - see the note about "The five pictures" (n ° 171 (ix)), part (b), "The formula of God."

760 (\*\*) Mebkhout has written in the form demonstration that  $m$  is an equivalence (demonstration on the same principle as that for the functor of God "analytical"  $m^\infty$ ) two years later, the end of 1980. This demonstration is presented in the second of two consecutive sections (the first deals with the functor of God analytical  $m^\infty$  resuming his thesis) "An equivalence classes" and "Another equivalence classes", in *Compositio Mathematica* 51 (1984), pp. 51-62 and 63-88. (Manuscript received on 10/6/1981.) But from March 1969 and in subsequent years he communicates result (along with the one on the functor  $m^\infty$ ) wherever the opportunity presents itself, and particularly from Deligne June of the same year.

I think because of its extreme isolation, and its "glasses" an analyst, he does not realize that it is especially functor good algebraic God would interest people like Deligne and others because it forms a "bridge" between the topology and algebraic geometry (pending arithmetic, I seem to be the first and only glimpse ...), a range comparable to that provided by the tool Cohomological spreads. Otherwise he would have taken care to make an essay in form

immediate and publish illico presto - especially considering the manners (he still did not know...) the strange environment in which he had

misguided. Yet his first mishap (with Kashiwara), in March 1980, it should have put a flea in the ear (s).

This is also the same in March that appears a note to CRAS Mebkhout "on the Riemann-Hilbert problem"

(t 290, 3 March 1980, Series A - 415), where it states the equivalence theorem of his thesis (for  $m^\infty$ ), and says cautiously

that "it is hoped show, using the method of cohomological descent as for the duality theorem [7] that functors  $S$  [I called  $m$ ]  $T$  and therefore [I called  $i$ ] are also categories of equivalences. "In fact, her demonstrations showed that they are equivalencies "locally on  $X$ ", which already implied, including, the famous theorem Kawai-Kashiwara (which will be discussed in the next sub-note), that the functor  $i$  (scalar expansion) induces equivalence between the category of  $3 \times$  -modules regular holonomic and that of  $3 \infty$

$X$  -modules holonomic. I note in

passing the final result of Mebkhout is considerably strong, even when applied to **modules** (instead of complex modules), because it says the same time as the canonical arrows

$Ex_n$

$D_x$

$(M, N) \rightarrow Ext_{No}$

$D_o$

$X$

$(M^\infty, N^\infty)$

from the functor "extension of scalars", are also isomorphisms (not only for  $n = 0$ ).

(X) (25 May) In a letter of April 24, Mebkhout precise besides me: "I must tell you that after my thesis I blew a little. It was four years I was under great pressure. "

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God (along with that of duality called "Poincaré-Serre-Verdier" in which the same Verdier absolutely wanted to believe three years before ...), from the mouth of Mebkhout more than a year after only the Bourbaki seminar June 1979 (four months after the defense). Still, that gives Verdier green light for Mebkhout present its results as status doctoral thesis, which he agrees to establish and chair the jury. If the defense is not made until a year later, it is because of delays Administrative imposed by the famous "Theses Universities Commission in the Paris region" (Verdier institution which holds the apple of his eye...).

As I said in a previous note 761 (\*), the defense is going on in indifference mood

General. Mebkhout has also beautiful send his thesis aplenty right and left, it continues to unnoticed - no deign to acknowledge the pavement.

Mebkhout yet does not let down. Despite evidence to the contrary, he feels a part, he a "Family" - people, after all, doing the same kind of math - those he has learned, in large part, attending my writings, and even more, by putting in opening provisions, listening with respect to a p. 1049 some **spirit** in these writings 762 (\*). He does not realize yet apparently not at the conscious level at least, that this spirit has long repudiated by the very people who form this "family" in which he believes to be entered, and that for these fine gentlemen who entered on mathematical high wool carpet, it is a layabout and an intruder.

#### **c2. the mafia**

**Rating** 171 2

(April 15-17)

**(a) downsides (family)** But Zoghman friend, who does not suspect anything yet and everything isolated as it is, is not unhappy. Since 1973 he has the chance to have a lectureship in Orleans, that her let the leisure to quiet the math of interest, and so what if this time they interested as him. He continues to live in the Paris region, to attend seminars to acquaint the literature. . .

If it was a little off on the thing, he would have realized yet that all was not well in this "family" who pretended to ignore it when he felt a part, he had come to realize account attending my writings, that much at least "good reference" that was for him like manna, was not of the growth of his "benefactor" Verdier. The concept of constructability was developed extensively in SGA 4 in 1963, twelve years before Verdier make mine to invent in this article. With the publication of SGA 5 in 1977, even in the form of publishing-killing of Illusie it 761 (\*) See note "... And the windfall" (n ° 171 (iii)).

762 (\*) One may ask (or wonder) what is this famous "spirit" so special in my writings that have inspired my "Posthumous pupil" Zoghman Mebkhout, and that would have been "repudiated" by all my other students, Deligne in mind, and by a fad

has followed suit. If I try to find a descent to this spirit (since my knowledge allows me more than parcel of the history of mathematics), I would say it is one in the line of **Galois, Riemann, Hilbert** . If I try to define it in terms of dynamic forces at work in the psyche, I would say it is a spirit that manifests itself a harmonious balance of creative forces "yin" and "yang", with a "base note" or "dominant" is **yin** , "feminine". A more detailed description of this approach to mathematics, and discover the world in general, emerges during the discussion in the notes "The rising sea", "Nine months and five minutes," "The funeral of yin (yin yang buried (4)) "(n ° 122, 123, 124) reflection in the recovery notes" Brothers and spouse - or the dual signature ", " Yin

the  
Servant, and new master ", Yin the servant (2) - or generosity "(n ° s 134, 135, 136) For a discussion on some rejection mechanisms "visceral" in the contemporary world, vis-a-vis this "spirit" see the two notes "The fact Providence - or Apotheose "and" disavowal (1) - or recall "(n ° s 151, 152).  
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appeared that this famous "bidualité Verdier" for complex beam C-vector analytically  
ment or algebraically constructible, was copied outright on the first set SGA

p. 1050

5 (the same one to which he referred in a volume with a strange name "SGA4 1

2

"By" various supplements

are given in SGA 5 I " 763 (\*)!). In this same strange volume, the author likes to express himself  
a superb disdain on the planet volumes SGA and SGA 4 5 around him, he could see a presentation on  
cohomology class associated with a cycle, which had relieved (it was not clear why) the volume  
"Technical digressions" SGA 5 (supposedly later...); he was able to realize at the same time as the appearance  
cohomological (dual of the homology appearance) of the theme which gave its name to the section of his benefactor,  
was also copied to SGA 5. For any of these three themes 764 (\*\*) in "good reference" there  
had however alluded to my person or 5. SGA. .

He could not know yet, of course, that what remained of Article Verdier (except three pages on cin-  
quante) was "pumped" out of my presentations on the homology of formalism spread and homology classes  
associated with algebraic cycles, missing exposed (by chance), and without even the trace of a hint  
their existence, publishing-Illusie of bleak memory, but the few facts available to him were  
certainly more than enough to put a flea in the ear of an informed and enlightened man. This was, in short,  
any situation similar to that which I had found ten years earlier, flipping Article Deligne  
on the degeneration of spectral sequences, where escamotait both the initial motivation and all yoga  
weights (and the role of my humble self), the contribution Blanchard@ideas, using  
precisely theorem Lefschetz "cow" for the fibers 765 (\*\*\*). Like me once, then had Zoghman  
silence the lucid perception of an unpleasant reality, saying (in this case) that there should be a  
"Connivance" of use between teacher and students, the teacher closes one eye when students are as  
their ideas, techniques, results they directly take him 766 (\*\*\*\*). As it often goes  
such cases, this interpretation (which suited well Zoghman) was not without an element of reality, which  
p. 1051

Furthermore. More than once, I had indeed been involved in such situations of ambiguity. (But he  
is also true that before I left, never before things had reached this point, where the work of the  
master becomes the remains of which we share shamelessly pieces. . . )

Moreover, in the more extended family formed of all those who are interested in the cohomology of varieties,  
including the Japanese Sato school, all was not so much for the better either. This same Kashiwara,  
whose 1975 constructability theorem was providential order to define the "good functor

God, "as it had pretended to attribute authorship of these unfortunate Plot beams that  
Suddenly everyone was tearing almost! He had renamed "finitistic sheaves" for purposes of the  
because, in s. 2 of the cited article, which contains more or less text message SGA developments to 4  
subject. From what I have heard from various sides, Sato school is familiar with my work cohomological,  
even though they quote me sparingly 767 (\*), and it is hard to believe that was not Kashiwara  
aware of the concept of constructability at least in the context spreads, which is the notion of finitude  
central to the whole theory. It goes without saying that a year later Verdier does not cite more Kashiwara to the notion  
"finitist" (sic), that breath word of a deceased or of a seminar 768 (\*\*). We may be the  
763 (\*) For this priceless understatement to ownership (by him, Deligne, this time) of the unfortunate theorem  
bidualité, see note b. p. (\*\*) Page 872 in sub-note "Trojan Horse" (n ° 169 3 ).

764 (\*\*) These are the "three areas": constructability, bidualité for Plot bundles, cohomology class (and homology)  
associated with a cycle.

765 (\*\*\*) for details see the beginnings of the note "Eviction" (n ° 63), and note b. p. (\*\*) on page 233 of this note.

766 (\*\*\*\*) (30 May) And while treating kindly humbug to boot. . .

767 (\*) Mebkhout writes about it (24 April 85): "The only references to you that I saw at Sato of the Japanese school concern  
Chapter III 0 EGA / while they were inspired shamelessly your work. ""

768 (\*\*) Coincidentally, this seminar (LMS 5) was that precisely (with SGA 4) which, by agreement between my students  
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beautiful world one and the other, and the same "family" may be why not - but when it comes to steak

vanity of authorship, everyone grabs for himself. . . 769 (\*\*\*) I think it was easier for Zoghman to p. 1052 that such Japanese he had never seen 770 (\*) was definitely "swindler", than having to see prestigious elders, one of which was for him as a powerful and distant father and benefactor, seniors he had the opportunity to work in seminars, and with whom he even had the honor of being with you and you (as is customary in the mathematical environment in France since the time of Bourbaki).

**(b) First trouble - or big shots from beyond the Pacific** Paradoxically, Zoghman of the trouble began when some people began to realize the power of the tools that was made in the wake of a philosophy (a kind yet that was as decidedly exceeded ...). He had told Deligne in June 1979, who had listened attentively to his explanations the duality theorem, and more (one suspects) the theorem of God. He even told her very kindly he had read the introduction of the thesis, and he thought there had to be in this working beautiful mathematics 771 (\*\*). Life was good for Zoghman, that day - but not for long.

The same year, in September 1979 he participated in the Symposium Houches 772 (\*), where he made a presentation p. 1053 cohomologists and in the words of their leader Deligne was intended to be "forgotten" (through the publication of digest-Kick saw his pen. . . ).

769 (\*\*\*) (May 24) Mebkhout tells me I blacked out a little table here. Verdier entirely ignorant Article Kashiwara as the notion of holonomy that Mebkhout taught him during his "interview" with Verdier in 1976. (This was before the publication of good reference (published end of 1976, it seems), but logically one can not expect it cites Kashiwara when he knows that both his colleague that he himself "pump" on the same unnamed source. . . ) Conversely, Kashiwara ignored the "good reference" and my bidualité theorem (contained therein under the paternity Verdier) is Mebkhout who introduced him to in January 1978, along with the results of Chapter III of his thesis. These were by Following the appropriate shamelessly (and practically without proof) in the article cited Kashiwara-Kawai - see this Note about "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), particularly page 1005. That was unaware Kashiwara bidualite the theorem for discrete coefficients shows, among many other signs readings here and there, how it was away from the philosophy of duality Mebkhout directly inspired by my work

770 (\*) (May 24) He had glimpsed once anyway, these famous Japanese! Mebkhout writes about it (22 April 85):

"The school Sato came in full force in 1972 for a conference on hyperfunctions. They hid well their methods. For a long time their results remained unaffordable. There was a certain mythology around This school, which is now Kashiwara can afford what he does. "

(June 4) It must be said that if it is true (as Mebkhout seems to suggest here) that the school would Sato initiated method to be surrounded by darkness in order to dominate, this process has found imitators of this side of the Pacific, which now are not left on their masters! And it is them, and not the Kashiwara and others, who have mounted the incredible mystification Symposium Pervert in which Kashiwara was used as a "pawn" convenient to prepare the ground - and then be dropped. . .

771 (\*\*) (3 June) Mebkhout had already got a compliment equally free, the previous year and from the mouth of Illusie the Symposium of p-adic analysis in Rennes. On this subject the note "Carte blanche to pillage" (n ° 174 4), 1091 page ( especially note b. p. (\*\*) page).

772 (\*) The Proceedings of the Symposium of Les Houches (1-13 September 1979) were published in Lecture Notes in Physics n ° 126 (1980), Springer

Verlag. In these Acts include both the presentation Mebkhout "On the Hilbert-Riemann problem", exposing all philosophy (I would call it the "coefficient de Rham") perfectly clear and references to support for demonstrations and a presentation by Kashiwara and Kawai. Any reader of good faith can verify, by comparing the two articles, there is no beginning of a philosophy of this kind, nor any mention of something like "Theorem of God", in the article by two authors.

(June 4) In its comment letter dated 22 April, Mebkhout is expressed in the same direction about the International Congress Mathematicians in Helsinki that had occurred the previous year (August 1978):

"I must say that I attended the Kashiwara conference was keynote speaker at the Helsinki conference (Aug 1978). There was no philosophy or far or near which may be akin to comparing coefficients discrete and continuous. I have taken care to write my illico Copenhagen conference which took place a week before and make available to the mathematical community that is supposed to be a judge. The conference this even Kashiwara is published in the Proceedings [Helsinki]. "

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"On the Hilbert-Riemann problem", with its equivalence theorem. His presentation seems Passer completely unnoticed. One of the "nails" of the Symposium, by cons, was a Kawai few conference days before announcing a remarkable and unexpected result obtained in collaboration with M. Kashiwara. Under a form somewhat convoluted and incomprehensible pleasure (according to the particular style developed by Sato school 773 (\*\*)), this theorem stated that a complex manifold (smooth), the functor "change scalar"  $3$  to  $3 \infty$  induces an **equivalence** between the category of three-Modules holonomes "regular singularities", and that of 3-holonomic modules. Their demonstration was going to be a very long section of more than one hundred and fifty pages, also published since 774 (\*\*\*). Mebkhout instantly, like all the other listeners, it was slightly dropped. This theorem, presented as



sensational and nobody really understand what it was exactly, yet had for him a "I whatever "familiar. In the days that followed, he stewed slowly but surely, in his p. 1054

habit. I can imagine that in the eddies of the Symposium, he has must have taken him a day or two, nothing that to put the theorem in a form understandable to a non-Japanese. From there, it was won!

I bet also that not a Western present had any idea what it is that these "sin-

Regular larities. "But Mebkhout him, he had already defined some years before, for the purposes of "philosophy coefficients" that still seeking a concept of 3-Module holonomic **regular** 775 (\*).

That one, at least it had a specific meaning for him - and taking the **derived class** suitable and passing more "on the other side of the mirror," he could interpret this category in terms of the derivative corresponding category sponding of "buildable discrete coefficients." At least he had shown far and wide in his

thesis analogous interpretation in terms of the same category of discrete coefficients "the other side"

category  $3 \infty$  -modules holonomic - and he knew he had to hand everything necessary to

prove also similar in the case "regular holonomic 3-Modules". That what he did in his thesis,

practically, as a result **local X**, which is already enough to involve the "sensational re-

Result "Kashiwara-Kawai. Thus, the perspective of derived categories, and that the game between coefficients

continuous, discrete coefficients, gave a result of the type of Kashiwara, Kawai, but in principle many

even stronger, since it gave simultaneously an isomorphism between  $\text{Ext}_i$  superiors, and not out ONLY

ment at the Hom (which was all we got, working with 3-modules without more, instead

Derived classes formed with such modules). This saw, it was the devil if this Japanese concept

"regular singularities" was not equivalent to its own - so the result would be prestigious

is a pure and simple corollary of his philosophy of the coefficients, which nobody previously had deigned interest is.

When the Conference at full strength just to honor with his presence the presentation of an unknown wave planned

the program no one knew why, and at the end of the conference 776 (\*\*) with arrows and

charts (the kind of stuff that were made in the sixties and were not long

p. 1055

for among serious people), this chap then ad without laughing that the famous "nail" the symposium (including person knew too would repeat the statement, which made it more impressive. . .) - that "nail", therefore, was

773 (\*\*) (4 June) On this subject a previous note footnote (footnote (\*) page 1052). Especially in the wake of the Colloquium

Perverse, it seems that the style of deliberate obscurity has been perfected to this side of the Pacific, a method

systematic deception and ownership to the muddle.

774 (\*\*\*) Mr. Kashiwara, T. Kawai, is holonomic Systems of equations microdifferential III System with regular singularities, Pub. RIMS 15, 813-979 (1981).

775 (\*) For the definition of Mebkhout regularity of a complex of 3-holonomic modules (along a Y divider), see

note "The work..." (n ° 171 (ii)), note b. p. (\*) 950 page "Regular" for short means: regular along **any** divider

(On all open).

776 (\*\*) (4 June) Actually Mebkhout had taken care to make it alluded at the beginning of his lecture, naively thinking that it would the gift of hanging his listeners.

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an immediate corollary of a category equivalence theorem (they ask you a little!) it would

obtained between the **derived categories** corresponding (what is that those animals?) and another

that did not seem to have much to do with them theorem that is contained in a **thesis** (the

bouquet that!) he swears he long ago sent to Mr. Kashiwara and many others among

distinguished colleagues in the large audience, it has the air of a bad joke. There is a silence

embarrassed, knowing smiles. This is (probably) to dispel the discomfort caused by the young lout, that Mr.

Kashiwara in person asks to use. He still looks a little stunned it must be said, it must

wonder if he dreams 777 (\*). . .The typical person, he does not let dismount either. It just that it will

not start a second conference over the first - we seen everything!

The next minute, our chap Zoghman finds himself alone in front of the blackboard with its beautiful

charts before an empty room. . . Nobody that day or the following days, has deigned to inquire about

ins and outs of the so-called "results" of the lout, we had the wrong to invite a Symposium

distinguished.

It still had to work in the head of Mr. Kashiwara, once past the flons-flons of

great occasion. Still, a few months later only, the seminar Goulaouic-Schwartz 1979-

80, in an oral presentation of 22 April 778 (\*\*), he announces **as of his own** this same theorem, which had p. 1056

had the gift of a chill at some symposium! Yet it was the "kindness" to add, on page 2:

"Note that the theorem is **also demonstrated** by Mebkhout **by a different path** " (that me

emphasis added) 779 (\*).

It "also shows" worth its weight in Kashiwara, while it is a theorem which he and nobody

doubted, and he had just learned (a few months before) from the mouth of the person himself, not having deigned to read the thesis that the latter had sent him for almost a year! If he had known before this theorem, for sure he would not have bothered to give a demonstration of 167 dense pages to demonstrate an analytical result "cow" which was an immediate corollary, even the corollary of a corollary. The "by a different route" is priceless. In the account in question there is not the slightest trace of a demonstration, nor indeed in any subsequent work Kashiwara or his Japanese colleagues, Zoghman assures me that there is no demonstration in the literature of his Theorem other than his own, and I doubt (see the kind of demonstration, which is familiar to me and because) we never found. It is a demonstration which corresponds to a geometric approach things, using the resolution of singularities to Hironaka - a tool that has become for me (and for my students) second nature, and analysts (including those of the school Sato) ignore. To such Point as Kashiwara obviously do not feel able to just **copy** the demo Mebkhout. . .

777 (\*) (4 June) Mebkhout writes in this (April 22):

"After the conference Houches someone told me it was even Kashiwara his article with Kawai was empty. But he has worked hard to catch up dishonestly. It has been five years [since his 1975 article constructability proving his theorem] he had not touched the discrete coefficients. His sudden celebrity [by this Article] due to any other problem allowed him to take care of things more "serious" - especially not bombinage! Enter 1975 and 1980 I was the **only one** in the midst of general hostility (something I understood after) to develop this philosophy child I learned in your writings. "

778 (\*\*) (4 June) Seminar Goulaouic-Schwartz 1979-80 presentation by Mr. Kashiwara of 22 April 1980, "Building Beams holonomic systems of equations and linear partial differential at regular singular points. "For details on this memorable seminar session where **Mebkhout was present** , see note "Carte blanche to pillage", n ° 171 4 .

779 (\*) I quote the statement of the written text, which was written by Kashiwara a year after the oral presentation. For details, see Note cited in the previous footnote b. p.

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This kind of scam sewn (very big) white wire can work, **as long as there is a consensus General which covers** the expense (here) of an unknown wave. All these people 780 (\*\*) would be wrong to interfere, while clearly said unknown is left behind by the very people best placed to know p. 1057

the facts firsthand, and have a direct and personal responsibility vis-à-vis the person: JL Verdier (chairman of dissertation committee) and P. Deligne (the first who felt the scope of the result he had learned from the mouth of Mebkhout the previous year).

Since I am kingpin overseas peaceful Kashiwara, so finish this chapter, with the epilogue the total elimination of the unknown service, the momentum of the striking example given three years Pervert at the Symposium in June 1981. This is an article by R. and M. Hotta Kashiwara "The invariant holonomic System on a semi-simple Lie algebra "(Inventiones Mathematicae 75, 327-358), published in 1984 (Received on 03.02.1983). This article, as it appears from the line 6 of the introduction, is one of many applications uphill "correspondence Riemann-Hilbert" called of God (or the unknown service). In this article, **the name of the stranger said is more pronounced** , and it does not appear in the bibliography. already knowing the mentality of the second author, but can not prejudice the bad faith of the first, Zoghman wrote to inform him that he was the author of the theorem there used critically, and to object the fact that it was not mentioned as such. Instead, the reference is to the paper cited Kawai-Kashiwara (167 pages), wherein said, theorem FIG moreover no 781 (\*). Hotta told him that he they had not seemed necessary to mention it, since in any case **it was well known that the correspondence in question was due to Kashiwara and Mebkhout** . Curtain. . .

**(c) The entry price or a promising young man** But Japan is far away, and if my friend Zoghman is slaving for years to break the Japanese distant center spears, it is probably because he was far more painful to him to assume the reality of a mafia which is not confined to the continents poles apart, but who has the hautdu pavement in both upscale seminars in Paris, in Moscow or p. 1058

Tokyo. It is time to return to the sweet country of France, and the "little family" formed by my dear former students cohomologistes, and (that, a little larger) that formed around them since the distant days of my "death".

The news travels fast sometimes. Current 1979 and 1980, Deligne and Symposium helping Houches, "We" had to end up realizing that he had just appeared on the market a mathematical theorem, my faith, promising, unfortunately due to a belated wave grothendieckien; but there was a subsitut all found at this paternity little exciting, in the person of the well known Japanese Kashiwara analyst, who asks to play the fathers of the famous "correspondence Riemann-Hilbert".

In January 1980, Mebkhout a presentation on his unfortunate theorem "seminar Singulari-

tees "Le Dung Trang, Paris VII. Jean-Louis Brylinski does not attend the presentation, but Lê Dung Trang him speaks and had him read his notes. From what he reported itself to Mebkhout when Brylinski acknowledges Mebkhout theorem, he exclaims, but with that, we will prove the conjecture Kazhdan-Lusztig! (Guess who was considered "unaffordable" as of right, by omens.)

780 (\*\*) (4 June) to a "parade" of actors who participated directly and actively spoofing scam-around the work of Zoghman Mebkhout (or at least those I@e read), see note "La Mafia" (n ° 171), section (f) "The parade of actors - or the Mafia." This parade is also not complete - for a more complete list (aligning names thirteen internationally renowned mathematicians), see note "The day of glory" (n ° 171 (iv)), note b. p. (\*) Page 962. We still lack the name of R. Remmert, appeared in the meantime (see note already cited "the mafia" part (c1) "The memories failing - or the New History ") - and fourteen (Besides an unnamed referred -... and fifteen)

781 (\*) (25 May) As already explained elsewhere (in "The five photos (crystals and 3-Modules)" footnote ° 171 (ix), see particular page 1005), the work in question contains a "half" only the theorem of God, half looted in Chap. III of the thesis Mebkhout.

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One might think that Brylinski will contact the applicant to be attributed to him in a more detailed mysteries of holonomy conditions and regularity, giving a precise meaning to the theorem he needed. But from what he himself has candidly explained Mebkhout "on" advised him not to speak to him, but to Kashiwara eminence. He did not specify who this "we". but obviously he had a good ear (in addition to a sharp mind), and it was then as unknown as Mebkhout is even just today. He did not say the thing twice, and he went to find out from Kashiwara, which was to be in the area again. That was his absolute right. The result was a joint article with Kashiwara, published by *Inventiones Mathematicae* "64, 387-410) in 1981 (received on 19 December 1980), with Title "Kazhdan-Lusztig conjecture and holonomic Systems". Brylinski was left star of the day next day, which was well-deserved, and Kashiwara added a flagship addition to an already loaded charts 782 (\*). Everything would be for the best in the best of worlds, but ... It seems that the same "we" had to p. 1059 also suggest that the less we speak of a vague unknown, the better it would be. There is always that in the manuscript sent to *Inventiones*, **name Mebkhout was not included** either in the text or in the bibliography.

Mebkhout had knowledge of preprint of the article, and he complained of the process to Brylinski, and wrote to R. Remmert, the editor *Inventiones*. Brylinski responded "flexibility" (in a style that is me now well familiar ...), adding on tests at the end of the bibliography (excluding alphabetical order) three references inch to Mebkhout (for that matter!), without making any reference in the text referred Mebkhout 783 (\*). A reader of this article, if by extraordinary sees the name of a famous unknown added late in the bibliography God knows why, will say that we had put it there to please a buddy. . .

**Brylinski has entered into stardom by a scam** . The truth is that the guesswork shows was unaffordable, as long as a new tool appeared. Regardless of the same **paternity** of that tool, nothing in this article highlights this new tool, whose role is retracted early (lines 6-8) by "explanation" (sic) neither fish nor fowl: "The method employed here is to associate holonomic Systems of linear differential equations with RS on the flag manifold with Verma modules, and **to use the correspondence of holonomic Systems and Building sheaves.** "

(My emphasis). There is not any reference or explanation about this famous "correspondence "unspecified." We "had to listen to the young first that" correspondence "was supposed become part of the well known of all things, for which there was no need to invoke p. 1060 a particular theorem, and thereby lift accessories paternity issues and (especially) premature. And Brylinski, a young man of the future, not the is not repeated twice. . .

As for Remmert, he forwarded the letter of the Unknown complainant referred to article Brylinski-Kashiwara. The referred dismiss the complaint, expressing the view that "the result **was known independently and pro-** 782 (\*) Involve the Kashiwara celebrity demonstration he had found, and where Kashiwara had had no hand while ignoring the crucial role played by his younger colleague unknown, was the "entry price" that Brylinski paid without being asked, for entering a "middle" of famous people - the middle which gives its name to this paper "The mafia "...

783 (\*) The introduction of the article Brylinski-Kashiwara ends with thanks expressed to various authors, including Jean-Louis Verdier (and without reference to the unknown service, is it necessary to say). She followed that with a par. One dedicated to

a summary of the "holonomic differential systems with regular singularities" (this is the Japanese name for 3-Modules regular holonomic). In the first lines of that paragraph, it says: "For the details and proofs, we Refer to the reader [6, 15-17]. "The reference [6] is Article 1975 of establishing its constructability Kashiwara theorem, while [15-17] (added on tests) is the "reference-thumb" to Mebkhout. The honor is safe no matter what happens for the "young man

**blement earlier by Kawai and Kashiwara** ", referring to the "theorem Reconstruction" he attributes to these authors (referring to p. 116 in the article cited authors, in the "Seminar on Micro-local Analysis" Guillemin, *Annals of Math*: studied, n ° 93).

This appreciation referred, which is supposed to know what he is talking, is scandalous to two titles, and shows that is an integral part of the same fraud in collusion with (at the moment) and Kashiwara Brylinski. It is already outrageous, on a simple **presumption** 784 (\*) are earlier results obtained independent p. 1061

ently (according to the same expressed by the referred opinion), to admit that the alleged post (as by chance the one that is unknown. . . ) Is not mentioned at all; such practices, obviously, open the door (and have long since opened the door...) to the most serious abuses 785 (\*). But there is more. The "theorem 784 (\*) (4 June) I even ignored here that this presumption was unfounded. The letter Remmert (from 01/26/1981) transmitting referred the answer does not expressly mention the date of the seminar Guillemin (quoted in the letter) and Statement

Kashiwara. I come in extremis to revive Mebkhout Italy (telephone ...) requesting clarification on this reference, and date. I learn that the presentation of Kashiwara is up in 1978, a few months after it has Mebkhout press Chap. III of his thesis (in January 1978) - Mr. Kashiwara has not wasted his time! As the defense thesis took place in February 1979 (due to the slowness of the unit represented by the Universities Theses Commission Paris, so dear to JL Verdier. . . ), This could give a plausible basis for the "presumption" of grandfathering referred, regarding the "Theorem Reconstruction" at least. But if the referred (in addition to being in good faith, which, already, is clearly not the case) had done his job conscientiously, he would have noticed that there is nothing like a **demonstration** of the "Reconstruction Theorem", in the statement quoted Kashiwara.

Mebkhout is also returned to the charge, in a letter dated 25.03.1981 where he points 1 ° ) that the theorem put forward by it referred was "one of the most important results of his doctoral thesis" and had communicated this, with its demonstration to Kashiwara (but forgets to say **when** - Zoghman never done in others), and 2 ° ) that this theorem was "Largely insufficient to establish the equivalence in question categories." R. Remmert has not responded to this letter, from a claimant nameless and support.

Zoghman me sometimes said (I will eventually know everything by dint of stress ...) he is aware of the scam Kashiwara Guillemin seminary the following year, in 1979, the year of his thesis defense. So this is his first confrontation with the kind of use in processes in "the mafia". At the Symposium of Les Houches, in September this year, so he knew already what to think about the big star Kashiwara. But like his philosophy and Results were written down on paper and published, demonstration and all, it would not never imagined it could be issue to retract his work altogether, once its importance is recognized. And the first sign of the power of his approach is just appeared at the Symposium of Les Houches, about Kawai-Kashiwara theorem. Of course, in January 1978, Mebkhout (which still had no reason then to be wary) had talked to not only Kashiwara what he called the "bidualité theorem" (later renamed "theorem reconstruction" for the needs of a scam) but also the theorem of complete good God, which was in fact a "half (the" half "the shallower of the two). He said that for bidualité theorem, Kashiwara was well "hung", it seemed that he had already wonder Questions like that; but obviously he had not the slightest idea how the show. (However, the demonstration Mebkhout does not use the resolution of singularities.) As for the theorem of God, it went off completely over head - so much so that he had completely forgotten the thing at the Symposium of Les Houches. Yet Mebkhout had sent him, like everyone, its complete thesis at the beginning of the same year (1979) (at a time so that he did not realized the scam Seminar Guillemin, the year before). Another thing that shows that the theorem of God had completely escaped the boss is that he has not even thought of the pocket and also for conscience for so to speak (even if he did not understand what was going on...), in the same presentation at the seminar Guillemin. Not having had the advantage so far to hold this presentation Kashiwara (\*) in the hands, I wondered if it was not not such as to justify the impression, in a casual reader, that philosophy developed by Mebkhout was known Kashiwara (and his own, as he says) as of 1978 at least. Zoghman promised to send me a copy of the statement in question, which, to me he says, allow me to disabuse me. There is (he said) an accumulation of

statements technical, more or less (in) comprehensible (Kashiwara least could do...), without proof and without thread apparent or anything (nor in its conference in Helsinki the same year, or that of Les Houches Symposium year after) that looks like a "philosophy of the coefficients" coefficients between continuous and discrete coefficients. (x) (June 16) Mebkhout says to me that the paper was in fact presented by **Kawai** , as a joint work with Kashiwara.

785 (\*) This is exactly the same attitude as that expressed three years later with the same cynicism, R. Hotta (in response to Mebkhout cited above): the new "rule", or rather "the law of the underworld" is to include people in positions of power (even outside the place) and not to mention the unknown (even though their contributions are crucial and attested by unexceptionable publications).

I do not doubt the good faith of R. Remmert on this occasion. But I see that as a publisher to inventors

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reconstruction "he cites (and which is also looted in the thesis of Mebkhout 786 (\*), where he appears under p. 1062 name (unfit) of "bidualité theorem") is still far from the equivalence classes (called "Riemann Hilbert ") used in the demonstration of the offending article of Brylinski-Kashiwara, equity due to one Mebkhout and does not imply 787 (\*\*). For me, the bad faith of the referee, relying on the connivence of cohomologiste establishment to boycott the name and work of an unknown wave the "benefit" famous people, can be no doubt. Anyone provided with a minimum of Culture cohomologico-Analyst, and a minimum of interest in a fascinating theme, can convince himself of the reality facts, and see a gross deception, to which the anonymous referred compete just 788 (\*\*\*). The situation is even less ambiguous than in Kashiwara or in any other Japanese specialists or other differential systems, the word "derived class" is pronounced until 1981 789 (\*\*\*\*), and again least there he has any reflection in the sense of a "philosophy" connecting discrete and continuous coefficients - p. 1063 philosophy which is equally absent in truth, vague references to the subsequent scrambles to some "correspondence (sic) between systems (resic) holonomic and buildable beams (reresic)" - No these fine gentlemen had this honesty until today, **only explicit black**

**white** (as I did earlier) **categories in the presence**, and the other to one of the arrows that establish their equivalence. By cons, a series of presentations of seminar notes and articles from Mebkhout 1977 attest to its pioneering work accomplished since 1972 in complete solitude 790 (\*).

I must admit that before being confronted with the thing, and have looked at and discussed at length and every angle 791 (\*\*), I never would have suspected, even in a dream, as a collective dispossession shameless can ever take place in the world of science. And it is a strange thing to have to tell me this iniquitous hoax was staged primarily by the combined two treatments among my most close old students; and furthermore, that the signal was given by **the appearance of a follower of my work** - a work in which I invested myself with passion, by putting what I had better to 792 (\*\*\*). After my departure, this work became the target and prey to the greed of those p. 1064 doubt) it has been misled by a dishonest referred. The referred expressed "hope" (cynical, given the circumstances) " **As a courtesy**, and Kashiwara Brylinski would mention the result Mebkhout". It was the role of R. Remmert, in As a publisher, to ensure that the result of Mebkhout is duly mentioned in the text, not as a "courtesy", but **respect for the basic rules of ethics mathematician profession**.

(30 May) Since these lines were written, I became aware of a new fact which throws an unexpected light on the role of R. Remmert in the scam around the work of Zoghman Mebkhout, showing its active participation in fraud around mine. So vanishes for me the presumption of good faith that I kept for him (for old habit, and failing irrefutable evidence to the contrary). The interested reader will find details of this "new fact" in part (c 1) (the note "The Mafia") follows, as "Fading memories - or the New History".

786 (\*) On this looting, see note "The five photos (crystals and 3-Modules)" (n ° 171 (ix)), end of part (b) ("The formula of God"), p. 1005.

787 (\*\*) See note cited above (Part (b) also) for the relationship between the "bidualité theorem" of Mebkhout and Theorem "The God" of which it is a half - the shallower of the two. It does not appeal to the resolution, while the Full theorem uses the strength of the resolution of singularities of Hironaka (which is a tool typically "Geometric", which was ignored by the Japanese school at least until the early 80s).

788 (\*\*\*)(30 May) and which R. Remmert, as publisher of Inventions, provides support without reservation. . .

789 (\*\*\*\*)(25 May) Mebkhout tells me that it is necessary to qualify this statement somewhat to the punch. While derived categories are virtually taboo in France soon after I left, the Japanese school continued to make a parsimonious use. This was a convenient technical means (to avoid the use of spectral sequences, in particular), but not the language "made on measures" to an intrinsic geometric vision of "coefficients" in cohomology of varieties and spaces of all kinds.

790 (\*) For a list of these items I need for me to go over here or even to enumerate, I refer to the article already cited Mebkhout and Le Dung Trang (in Proceedings of Symposia in Pure Mathematics, 40 (1983) part 2). (25 May) See also bibliographic references in the pages in the note "Three milestones - or innocence" (n ° 171 (x)).

791 (\*\*) (1 June) I did it first last year, in the week of May 2 to 9 (writing the "Procession VII", named "The Symposium - or bundles Mebkhout and Perversity"), and again for nearly two months, writing "The Apotheosis"

792 (\*\*\*)(In this page retyping net (quite heavily crossed out), I thought is coming if my investment in this

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same that were closest to me, and a secret violence, beyond my person and my work still strikes those who openly were inspired. . .

(c1) **Fading memories - or the New History** (30 May) Six weeks after

wrote the preceding pages, I pause here in the story of the misadventures of my friend Zoghman, for some dwell on the "new fact" which is alluded to in a previous footnote page (note 793 (\*) 1061 page). We can read the following pages as an interesting supplement the flowering of the "new style" which discussed elsewhere (\*), which style excels in the art of writing (To the satisfaction of all...) A "New History" (a certain theme of contemporary mathematics, as it happens...). The hurried reader to know Following the misadventures of my friend Zoghman (misplaced in a circus he could not have envisaged) can continue directly with "The General Repetition (before Apotheosis)" (part (d) below, dated 16 April).

I have read the introduction and bibliography of the book "Not Archimedean Analysis" by S. Bosch, U. Guntzer and R. Remmert 794 (\*\*). This book presents the theory of rigid-analytic spaces, having rightly notes ("private") J. Tate of 1962, "Rigid-analytic spaces", as the point starting from the theory. It is stated in the introduction that R. Remmert "could get a copy" of this rare document, which had represented somehow the Birth Certificate of a newcomer the Areopagus notions of "varieties" (analytical, case).

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Remmert had forgotten that it was I who had taken care to multigraphier this document care the IHES (which formally started) and to make him a copy, and other specialists complex analytic spaces - history of drawing attention to this unexpected extension of their theme predilection. This was at a time when none of them even pretended to be interested in the basic body other than real or complex - but you never know. . .

Remmert had to forget also that if I was so interested in such a point spread among my friends that text confirming the emergence of a "universe" new geometry is (among others) because I had been associated with Around this birth. The very name of rigid analytic space was found by me before Remmert

no one (not even Tate!) have heard the name or have only dreamed the **thing** that this name was Express. I was the first to see the "rhumb" theory of elliptic curves Tate as before

be a "passage to the quotient" to a kind of "analytical" varieties that do not yet exist, and which should lead to algebraic-analytical comparison theorems like "GAGA" Serre. There was a

Another motivation that showed me the way to the same type of new objects: the need to define a "generic fiber" for formal schemes of finite type over a discrete valuation ring.

work focused (among others) such fruits, unexpected and unwelcome, it is probably that this investment in himself and in the spirit that animated me, there was not this "best of myself" as I like to point out here, but there was also the "worst". This is something that had appeared quite clearly, certainly, in Fatuité and Renewal (first of Crops and Seeds), but also something that egotistical mechanisms of great strength push me constantly to forget! I begin to realize that this "worse" was that **glimpsed** in the reflection of last year, I have not done a really thorough examination, or "tower" that reveals me to really detail the various faces. This is why the knowledge I have is superficial, like the action of this knowledge (in my relationship Burial, in particular).

This fourth part "The Four Operations" Crops Sowing especially represents a working recolement méticulous of **facts** gross related to Burial. This work "stewardship" has nevertheless helped to make me feel that an understanding further to the Burial does not come as me of the kind of work I am doing for nearly three month simpleton of a deepening of the work done in Fatuité and Renewal, that is also: a deepening my knowledge of him that I was, in those distant days of "before I left." . .

793 (\*) See note "Congratulations - or the new style", n ° 169 9 .

794 (\*\*) Grundlehren der Mathematik, n ° 261 (1984).

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As third indication pointing in the same direction: I had heard that Krasner (well known in the fifties and sixties in the Parisian mathematical circles as an original that housed home an army of cats, and walking in all seminars with his big coat and Russian its always beaming. . . ) - that Krasner therefore "made of analytic continuation" on bodies not valued Archimedean. I do not know anymore and I'm not sure I met someone who had read the work of Krasner on this issue - but the thing was enough to intrigue. It must be said that the term "analytic continuation" did not by itself virtue to break my heart stronger (rather the contrary, it reminded me memories unchallenging of my student days. . . ); but once glimpsed the need for a new type of geometric objects, it could only make tilt. . .

Going back to Remmert - if his memory is so faulty item, original text of the Tate (be prides to own) could nevertheless refresh him. In his notes, Tate makes no secret of the role I had played in the development of the theory 795 (\*), writing among others (I quote from memory) it followed "to p. 1066 way fully faithful "a supervisor (for the concept building process" glueing pieces ") he was holding me. I had him again provided some type of" building blocks "(or of "location method" in restricted formal power series algebras), for the purposes of the fibers

formal schemes. He had completed the first "pieces" (or "processes") with those of a second type, somehow complementary.

This new concept would probably not see the day (no more than étale nor cohomology nor crystalline nor many other things that followed in the wake, including even the last "pie Cream", the famous 3-Modules...) if I had not had the common thread of "generalized spaces" (now thereafter **the topos**), whose theory remains to be done, but was already foreseen for four years. It is this intuition that showed me the way to a type of "varieties" that precisely **out** of context spaces topological (locally annealed) common.

From the moment the **local theory** of rigid-analytic spaces was started by John Tate, it is me who also asked and popularized the statements of theorems crucial first "global" to prove about these new varieties, statements that had been present in my mind right before a first working foundations be fulfilled: theorems of algebraic-analytical comparison for own diagrams relating to a rigid analytic space, finiteness theorem for  $R_i f^*$ , for a morphism own  $f$ -rigid analytic spaces - problems solved by Kiehl in the years following 796 797. But it is true that following the wind blowing today, it is regarded as something unimportant, p. 1067 795 (\*) More than twenty years have passed since those distant days, when a close friendship linking us, Tate and I and his family and mine. For years I have received life-sign him. I did not have either knowledge that it is moved, nor any other among my students and old friends who could not fail to take notice of this book of the retraction of me that is made in the introduction. Other times, other manners. . .

796 (\*\*\*) I note that as soon as Tate laid the foundations of a theory of rigid-analytic spaces, it was clear to me that the context in which it was located was still provisional, and in no way exhausted the intuitive content I tried to express in the name of "rigid analytic space" - any more than the type of schemes over a body exhausted intuition associated with the word "scheme". A common thread to a substantial widening of the context Tate (I put in prior to anyone who would listen. . .) Was provided by Tate himself, who wrote a "universal Tate elliptic curve" on a some topological ring (the subring of the ring of formal series  $Z[[t]]$  that are convergent for  $t$  in the disc open unit in the complex plane, if my memory is correct), which ring obviously had to be considered "the ring of affine coordinates" a rigid analytic space, of a type which does not fall within the range offered by Tate. Given the general contempt in which fell soon after I left, all questions of fundamentals, it is not surprising that the conceptual apparatus developed by Tate in 1962 has not moved a muscle since.

797 (\*\*\*) (4 June) I was also the first to insist on the need to introduce for-rigid analytic spaces, "points" more general than those considered by Tate (with values in extensions **finished** only the basic body). This need was suggested both by analogy with algebraic geometry, by the desire to find a concrete interpretation "points" of the topos associated with the rigid analytic space considered.

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and ultimately, just humbug, than providing new concepts, to identify prime contractors, and ask the questions that real mathematicians would solve. . .

Anyway, my name is not pronounced in this introduction, to have something to do with rigid-analytical areas. He Krasner either, for that matter - on the contrary, the theory of Tate is presented as introducing "a rich enough structure to make the impossible possible: the continuation analytical on totally disconnected body" - so that in 1962 said analytical continuation ("impossible") was already ten years, if not twenty or thirty (I can not say), the "name" official (So to speak) of Krasner. No Krasner either trace or me in the extensive bibliography. My Yet name appears in passing towards the end of the introduction, in the name "Grothendieck topologies"; for This concept is referred to notes Artin (1962), in superbly ignorant (following the example set by the cohort of my former students in full force. . .) Development of meticulous work done in SGA 4 (since 1963 and throughout the sixties, but in a noticeably unwanted fatherhood. . .). no mention either, not surprisingly, the role I assign to rigid analytic spaces in the development of crystalline cohomology at a time (1966) where Remmert (no more than any of its distinguished colleagues Analysts complex) still showed the slightest inclination to be interested in these funny (so-called) "varieties", say "stiff-analytical" (just asked you...), which had been concocted in some corner of their algebraic geometers - like complex analytic spaces were not sufficient to fill leisure analysts and surveyors seriously. . .

Just be informed first hand on the true story of the genesis of the theory presented in p. 1068

the book, to see how spreads in this introduction the same cynicism that was also expressed in the response by an anonymous complainant referred to a stranger (with the blessing of the same R. Remmert) obviously, in the minds of the authors, it is a simple matter of "courtesy" yet, a "kindness." In short, they are free to grant or deny, if they go or not to include in their "historic" (sic), the name of such or such who had played a crucial role in the genesis of the new theory. For them (as Also, should we believe, for almost all of the mathematical establishment, who cashes it without flinching kind of falsification. . .), The "History" is not **what actually took place**, but is something that can

be **decided** sovereignly by the person who claims the right to write it, or the consensus of a handful of people who decide what happens to be, as to what happens to have been. These people love to make hot sips of what happened and is still happening in the Soviet Union, and not a louveront (I know whereof I speak) to sign manifests for the "defense of freedom" (of thought and all that...) **among others**, while exercising the same dictatorship of the lie, where it is **them** who have power.

(3 June) Evoking the previous pages, there are a few days, the picturesque figure and endearing Krasner, came to me the question whether he was still alive. He was my elder generation or two, and it felt like an eternity (although fifteen years, if not twenty) that I had not heard him say his name. While I strongly remembered character, it took me a few seconds yet before I return his name. (It is true that this is the kind of thing that I often now, helping age. . . ) Krasner had to be very hospitable reputation and his Russian origins were another point common that could bring us closer. But I was too stuffed into my math for availability to link me to just friendship "for fun". Our approaches to math to be sure poles apart from each other. We did have a chat together once or twice, between sessions of a Bourbaki seminar if it is, but not math surely. And there was little that math so that I really hung. . .

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Still today I get a note Deligne, just a few lines on a matter p. 1069

practice inconsequential story perhaps to remind my good memory (must be a few months there was no letter of exchange between us); and also to place a postscript, I beg to reproduce here (assuming the agreement):

"PS I was sad to learn that Krasner died there two weeks. I

always reminds of an exposure river he gave in Brussels, there are about twenty years, which of course went over my head, but I stayed a the last few listeners.

It struck me that it does not appear in your picture fifties 798 (\*), where he made beautiful things - even though foreign to the spirit of Bourbaki, and a genius for evil definitions flared. "

So here another Praise of Death, for one of my co-buried this time. In it I think I see conveyed a sense of sympathy, or perhaps a reflection of such a sentiment that had once been alive. But just as in my praise of Death, my friend Peter did déserrera teeth to say, in honor of this Once a disappeared without return, **what** were these "good things" which he likes to refer without the appoint. Yet he knows as I do that these "things" have prepared the advent of a theory today full bloom - and for reasons he may know, the New Masters have more to bury prematurely (and with me) this precursor good child, draft and "evil flared" which has just disappeared; one, surely, who "made the analytic continuation" on ultrametrics body, at a time when Tate Remmert or I "did" yet the case of equality of triangles and the Pythagorean theorem, and when the friend Peter was still nose (and wipe...) By his mother!

**(d) The General Repetition (before Apotheosis)** (April 16) But I must return to the series "Misadventures" of not stitched to my posthumous student Zoghman Mebkhout. I have no idea what happened in Deligne head in June 1979, when he learned from the mouth of an unknown wave is p. 1070 claiming Grothendieck ideas, elegant solution to a crucial problem 799 (\*), on which he had striven decade earlier for a whole year without reaching an answer that satisfies him. Given its provisions long, we suspect that he was not going to congratulate the young man to have succeeded where he, Deligne was failed. But I have the impression that its provisions are so gravedigger failure point flair (as I knew surprising) that either did not hear him even now (after six years), the true scope ideas and vision of the unknown wave. Like everyone else, he saw finally that "cream pie" unexpected tool that everyone expected, iron fracturing "problems proverbial difficulties." A day, however, he had endorsed a comprehensive vision that another had communicated - to bury and vision and one in which she was born, and grab a tool still turned also in "Iron fracture". . .

The first trace that I may be known of any reaction to Deligne theorem is Mebkhout a short handwritten undated letter to Mebkhout letter received October 10, 1980 800 (\*\*).

798 (\*) Here there is a clear misunderstanding of my statement in the first part of Crops and Seeds, "and Fatuité Renouvel-LEMENT. "At no time was it about to paint a" picture of the fifties "mathematical, if only that of Paris medium or that formed around Bourbaki. My main purpose was to discover my past mathematician. This is what led me to talk about my relationship with such colleagues or students, when it appeared as important in my life, or could enlighten me about myself.

799 (\*) (25 May) It is possible that Deligne had long since lost the sense for that character "crucial". On this subject the note "... And hinders" (n ° 171 (viii) ©

800 (\*\*) This is the document "statement under the seal of secrecy, and which I here say a word...", Which was discussed in



Note "Victim" (page 309). Looking back a year since, Zoghman has kindly allowed me to reproduce right here.

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"Dear Mebkhout

I sent Bernstein and Beilinson my copy of your thesis: they need your results for their proof of the conjecture Kashdan-Lusztig (I have an abstract, in Russian, of their work, I will send you if you want). Could you send me another one?

Thank you.

P. Deligne "

I assume from this letter that Deligne had to inform two Soviet mathematicians on the Theorem of God, perhaps suggesting that it could be used to prove the conjecture in question; either that he be realized by itself, is that the noise was already running that would Brylinski ideas on this subject. The presentation of Mebkhout that had "tripped" Brylinski was already in January 1980. Articles of Brylinski-Kashiwara one hand, Beilinson-Bernstein on the other, proving the famous conjecture using the unnamed even less of an unknown named theorem, have been received, one December 19, 80, the other p. 1071

December 8, 1980, so eleven days of each other. Mere coincidence?

The thought even came to me why Deligne, who had knowledge of the new tool before all others, in June 1979 (since nobody, including Deligne, had bothered to read the pavement of the wave unknown) - why Deligne has not thought himself to apply it to this conjecture, and reap so new laurels instead of helping his Soviet colleagues to pick? He does not mind though less bright Brylinski that? It could be that from that moment he saw the possibility of recovering by a band Paternal theorem of God himself, who (as he had the feeling) should have been hers since Ten years already; it was a sort of unacceptable maldonne this young uncouth presumptuous had assumed the right to prove things on which he, Deligne, had already addressed long and unsuccessful conclusive. He had finally failed him just a bit to get there, it was not just another harvesting where, he had sweated in vain. . . But if he wanted to recover that, in fact, was his by right (According to the unwritten law that eventually prevailed in a high-class world he feels the center and the boss ...), he had to maneuver with a different fingering, and not try too to swallow time 801 (\*).

Still, that Zoghman, already smarting from the strange episodes with Kashiwara and Brylinski, Judge prudent to inform himself MN. Beilinson and Bernstein theorem which Deligne said they needed - both of that so as Mr. Deligne forgot to remember, talking them theorem, which was the modest author. It was fortunate: the following month, on 24 or 28 November 1980 there Moscow was the "Conference is Generalized Functions and their Applications in Mathematical Physics" to Moscow. Mebkhout will give a presentation on his theorem, published in letitre "The Riemann-Hilbert Problem in p. 1072

Higher dimension "and he is careful to talk to Beilinson and Bernstein in person to explain in detail the ins and outs of the result.

It came at just the right time. It©just ten days after the conference that the two authors fend their work on Kazhdan-Lusztig form of a note to the SARC (t 292, 5 Jan 1981, Series I -. 15), "Theory 801 (\*) This is of course a simple presumption that the purpose of ownership on the famous "correspondence" was present from the time Deligne has read. I for one am convinced. It is true that the letter quoted above seem give a presumption to the contrary. I personally see the sign yet of a challenge - that he, Deligne had absolutely not paying attention so little it is, the time it was an unknown wave, **which does not move, all ways** , while he was alone against all; that he, Deligne, could afford to "commit himself", as he could also allow, for the provocative name "perverse sheaves" to proclaim, symbolically, yet brilliant, the true nature of its provisions. On this subject the note "The perversity" (n ° 76), and (in a psychological context enough similar but less extreme) the note "The joke - or" complex weight "(n ° 83).

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Groups - g-module Location "Note Alexandre Beilinson and Joseph Bernstein, transmitted by Pierre Deligne. As just the name of Mebkhout was not mentioned on their manuscript - apparently Deligne had entirely forgotten to tell them about the unknown wave, he started off well their com-qué the thesis, for just ...? Figure it! Mebkhout comes with great difficulty in convincing Beilinson ( "the most honest of the two" me-he says with great seriousness of the world) and in Article Kashiwara-Kawai they quoted in the literature, there is anything but "construction" (replacing here Uphill "correspondence") which too, like everyone else, speak only by allusion, (surely

Deligne, while communicating their thesis of the unknown where the desired result was indeed <sup>802</sup> (\*), has owed to suggest that perhaps it was more reasonable, if they wanted to give a reference to cite item Kashiwara and no matter the background which, as no one would watch so closely. ) Are promised when even stranger audit appeared there in person, we would think of him and that would rectify the shooting for Kashiwara. Sorry - the story of the misadventures of my friend is decidedly Zoghman repetitive! In the note of these brilliant writers, **transmitted by Deligne** (which I have reproduced the letter, written a few months before) **Mebkhout name is not pronounced** . He Kashiwara not either (and I already see there point a piece of an ear ...). There are cons by a double reference on the run in the last part of the note (proving Kazhdan-Lusztig) to a " **building set forth in** [4], [5]..." <sup>803</sup> (\*\*), "construction" which (you p. 1073 guessed it!) is none other than the functor never named the unknown service, much less named. The reference [4] is an article Kashiwara (provisional surrogate father). In this article of course (not more than in Kawai-Kashiwara, which goes to profit and loss), there is nothing that closely resembles or far to the "construction" as reported by these authors; This article is also of 1975 <sup>804</sup> (\*), so almost five years before the presentation of an unknown wave at Les Houches Symposium gives this same idea Kashiwara it would not be so stupid after all to utter the word "derived class" and appropriate way, according the simple law of the strongest, the credit for the labors made by others. As to the reference [5], it is exposed Mebkhout of the Les Houches Symposium in September 1979 - the same one where Kashiwara learned that derived categories, it could be useful, and something else to rip an unknown left behind by his bosses and seniors. . . .

<sup>802</sup> (\*) (April 17) There was at least in theory a very similar result, even if the version in the form used by Beilinson-Bernstein (and Brylinski-Kashiwara) was not included in full. See Note b. p. that same day (footnote (\*\*)) page 1047) for details.

<sup>803</sup> (\*\*) We admire its value the wave of "building **exposed** to...", Leaving open the question completely which is due to this "construction" (or "match" or "relation"...); that issue will be resolved with the virtuosity that known six months ago, during the famous Symposium (see note "The Conjurer", n ° 75 ") will be taught there in the article Beilinson-Bernstein-Deligne, the terse reference [4] [5] (in two places, surely, construction should be well (Luckily) "exposed") was pure courtesy, and the brilliant father of the "match" is the one we guess. . . But even apart from the conjuring trick that I just mentioned, it is already a scam in itself as to refer to a new theorem, deep and hard by the term "construction exposed in ...", as if it was a simple "Construction" precisely who allegedly dragged there by sheer chance and the perpetrators would have chosen, for the greatest of chance also to use here for their brilliant demonstration. I recognize the same spirit as that of the operation "SGA4 <sub>1</sub>

2 - 5 APG ", which consisted for recall (passing)" exposed construction "in APG APG 4 and 5 with a formalism cohomology spreads (and the "matrix of non-sense" whose brilliant author was forced to extract), before mine up his sleeves and start making " **real** math...."

(May 25) See, on this "new style" note "Congratulations - or the new style" (n ° 169 ♪).

<sup>804</sup> (\*) made verification, it is Article Kashiwara already cited, where it demonstrates its constructability theorem, which plays well secure a crucial role to define "functors of God" (functors that person yet except Mebkhout never had dreamed before the rush of 1980). This is a gross fraud that pretend to confuse this theorem of Kashiwara (that nobody thinks to question it) with the theorem of God, incomparably deeper, and a different scope. Demonstration perspective, this theorem uses the power of resolution of singularities to Hironaka. From the point of "Philosophical" view, much more importantly, it builds bridges that were missing in the cohomological formalism between the topology, algebra and analysis (pending arithmetic, though some I see end up finding fossoyant the use of their healthy schools. . . ).

817

No more than in the article Brylinski-Kashiwara, anything that would give the slightest hint at a player p. 1074

that would be really good "in the know" that this brilliant score would not have seen the day without the apparition of a new and providential tool retracted under the euphemism "construction exposed in ...". I

also recognize the proven <sup>805</sup> (\*) flooding of a fish, called "dilution" by "docking"

the person he is retracting (though we like to be "go" yet and we need to say

the city. . . ) With another that has nothing to do with the issue or whose role is minimal, as if to say

here (between the lines, and yet very clearly) that unknown wave that put it there (out of courtesy and given its insistence) has no more to do with the famous "construction" (including the new consensus

control came to speak only through allusions and as a well known thing of all. . . ), Such section

appeared in 1975, at a time when nobody in the big world still deigned pronounce the word "category derived" (if not just as a joke...).

**(e) Market fooled - or puppet theater** I do not regret taking the time to

my benefit as much as for that of a mathematician player who would be interested in the thing, having spent here through the three preliminary scams around the theorem of the unknown service. these scams

are raw Kashiwara to Brylinski-Kashiwara (with the assistance of a referred remaining anonymous), and

Beilinson-Bernstein, with a Deligne in the wings <sup>806</sup> (\*\*). They reflect a uniformity of style striking, on which there is no need to dwell further. This is the style I have read ad nauseam throughout my long investigation into the burial <sup>807</sup> (\*\*\*), and is prefigured strikingly in the 1968 article of my most brilliantly gifted student, the same Pierre Deligne <sup>808</sup> (\*\*\*\*). And this circumstance also enough to remind my good memory as an ambiguous attitude and complacency p. 1075

vis-à-vis Deligne and others, I saw brilliantly gifted, I am not without having contributed my share corruption that I see spread everywhere today.

It also becomes clear that the apotheosis of Perverted Conference of June 1981, just six months after the third episode we just go over, do not fall from the sky. Strangely enough, this conference was (to my knowledge) the first and only after I left, which was devoted (to say the course, and yet unequivocally) to exhume a component of "mathematics grothendieckiennes" for the occasion unexpected suddenly appeared a new tool, which has proved irreplaceable. This tool that was livable an approach things the consensus of fashion had long ago as rows as outdated and vaguely ridiculous <sup>809</sup> (\*). And by a strange turn of events, due to the particular genius My former student shining, this striking confirmation in practice and under the pressure of needs, a approach disowned by him and by all, as was the opportunity, through the same Symposium, from burial total and definitive of the deceased master and not appointed by the posthumous student company (equally unnamed) who had had the good fortune (or misfortune...) to move all these people.

<sup>805</sup> (\*) For other examples of this so-called "dilution by assimilation", see sub-note "The real math..." N ° 169 5 ) Note b. p. (\*) Page 885.

<sup>806</sup> (\*\*) (June 5) The role of Deligne "behind the scenes" is clear at least in the third episode, and there are strong presumptions in the same direction for the second. But it seems that Kashiwara has "fired" (for scams around the work of Mebkhout) for its own account in 1978, at a time when so (apparently) Deligne was even aware You@ welcome. On this subject part c) of this note ("The entry price - or a young man of the future"), note b. p. (\*) page 1050.

<sup>807</sup> (\*\*\*) See about this style, including the end of the note quoted above "Congratulations - or the new style", n ° 169 9 .

<sup>808</sup> (\*\*\*\*) See the beginning of the note "Eviction" (n ° 63).

<sup>809</sup> (\*) For the psychological mechanisms at work behind this "consensus mode", covering with some "reaction vis Cerele "rejection front of a mathematical approach to style, see notes already mentioned" The providential circumstance - or Apotheose "and" repudiation - or recall "(n ° s 151, 152).

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## 18.5. THE FOUR OPERATIONS (a corpse)

This conference did not fall from the clouds, no. A feature among many of my friend Pierre Deligne is that he knows to wait and seize the right moment. The three episodes around the "cream pie" with the almost complete elimination of any mention of previously unknown service, showed him the evidence that the time was ripe for picking discreetly and with the smiling and affable nature that characterizes it, which anyway was supposed to return him right. I presume that there has been careful consultation with Verdier which he had to understand that it was time to exhume large bands categories dérivées and "paternity" long repudiated; at the same time, buried under the spotlight and the unknown wave, and the master has long deceased (of time someone would have the wrong idea remember that it was something to do with all these beautiful things were suddenly figure "trendy". . .).

Kashiwara as father-to-the-sly of a theorem-the-good-never-God-appointed, it was well p. 1076 for a moment ; just as long it was understood that we would neither name nor write Theorem in question. Kashiwara itself was not too hold them responsible, to this theorem which he still included less if it is, Verdier in person - he had the pocket passing as inadvertently and the opportunity of habit. Deligne him who waits, knew that this theorem was not going to stay forever theorem no address and no name. It was, in short, a theorem **in search of a father worthy of him** , and that would be able to appear in the full light of day once the "real" paternity, which normally should have been hers (and for twelve years already ...), would be the subject of consensus General and intangible. The article "perverse" jewel of the same name Symposium was a first step in this meaning, posed by the principal party with his usual skill.

I feel that Beilinson and Bernstein probably flattered to be associated unexpectedly fatherhood on said beams, too (but wrongly) pervert, and a guy prestigious yet, were actually manipulated by Deligne in order to serve as his alibi "in case..." As the article is written, any reader that would be very well informed can only think that it is none other than Deligne, of course, who is the author the providential "match", though never named or stated in clear (because everyone is already supposed to know. . .).

This leaves just the shadow of ambiguity (carefully calculated) in this great turn of phrase about "the relationship" non.nommée which "ought to find its place in these notes..." <sup>810</sup> (\*). That was how

"thumb!" to listen carefully and clearly, without saying in words, as said relationship (if any noted) was due at least in **one of three authors** of the brilliant article, or (at the utmost rigor) to the three together. But it was also clear that when the time (for the those who wait. . .), It would not be Beilinson, Bernstein nor who would argue a paternity already Deligne almost acquired. There had to be a market <sup>811</sup> (\*\*), tacit if not expressly stated: at Beilinson and <sup>p. 1077</sup> Bernstein conjecture Kazhdan-Lusztig and (for good measure, given that there was already Brylinski-Kashiwara <sup>810</sup> (\*) See note on this subject already quoted "The Conjurer" (n ° 75 ").

<sup>811</sup> (\*\*) The presumption of such a "market" came to me by association with two similar situations. On the one hand the market (perhaps be implied, but clearly apparent) between Deligne and Verdier, this "sacrifice" Lefschetz-Verdier formula that goes to profit and loss for "business purposes SGA 4 <sub>1</sub>

2 - 5 SGA "but" picking "in consideration all the inheritance" duality " the deceased, and derived categories (Article discount) bonus. (For the detailed story, see subscores group "The Formula "n ° s 169 <sub>5</sub> - 169 <sub>9</sub> .) Moreover, there is the "market" concluded by Deligne said with a dead master, who had in any case disappeared from circulation and was not likely to react, about 7 seminar SGA is shared during two years from 1967 to 1969, which was "shared" by three years of half and half, one for the deceased, and the other for a Deligne makeshift teammate. (For details, see p. Ex. "Episodes of climbing," said n ° 169 (iii), episode 2)

It also associates with the "market" with the same deceased (do unsuspecting) for the said guess (MacPherson dixit) "Deligne-Grothendieck" (see episode 1 in the same note already cited): the first half for "factor" that Deligne MacPherson was informed of a conjecture (kept secret until now by the care of my cohomologues students), and second for the deceased, in his capacity as "collaborator" of the first. . .

819

above) co-authorship on said beams (by agreement, I guess) "perverse" <sup>812</sup> (\*); to the Deligne famous "relationship" nameless, waiting for the day will be soon and without modesty having to disturb, where everyone will call "theorem of Deligne". And the future "father" was pretty much the end nose to know at least this, about this child (he had repudiated once rather than consent to accou-expensive. . .) That concluded there a "good deal" <sup>813</sup> (\*\*). As for Kashiwara, his role was over, and it is no more mention of him in the brilliant article about the providential "relationship" as the unknown service. All against one when it is an unknown wave, okay - but once cleaned instead of an intruder, every man for himself. . .

**(f) The parade of actors - or the mafia** The "family album" opened there three weeks hardly <sup>814</sup> (\*\*\*), comes unexpectedly to enrich a few new faces. The "family" has greatly enlarged die, obviously, and I@ crumbling has trouble recognizing it, especially as times have exchange. This time and by order of appearance, it was **Mr. Kashiwara, R. Hotta** <sup>815</sup> (\*), **JL Brylinski** and <sup>p. 1078</sup>

the **anonymous referee** of the article Brylinski-Kashiwara to Inventiones. A group of "hard", that@for sure, the reflexes running smoothly, and most agree the finger and the eye when it comes to scam a particular wave, a discreet sign on the Grand Chief in the wings (or even without waiting for a sign...). And again I find the appearance of a **mafia** <sup>816</sup> (\*\*), reigning supreme in their undisputed fiefdom whose heart is the cohomological theory of algebraic and other varieties. Shiny, hard people with brains immaculate, I have revised the work over the course of four successive episodes of the so-called operation "of unknown service ", culminating with the Symposium Pervert. In addition to the four guys I just mentioned (including anonymous), I remember the good memories of the five other members of the "hard core"; it actually nine that have mobilized to bury **the intruder one that is not of their** .

There is the Grand Chief, **Pierre Deligne** - one who always knows "wet" the least, while pocketing the more. There his second, **Jean-Louis Verdier** says "benefactor" - the same one who presided over the jury of certain thesis of a stranger, and he still was one of the organizers of a memorable Symposium robbing shamelessly that even unknown. There is the other main organizer, **B. Teissier** , who signed jointly with his memorable introduction to memorable Proceedings of Symposium memorable. Unlike the others, it seems that he simply acted sidekick and nominee, while <sup>p. 1079</sup>

had nothing to gain for himself - if not the only fun to be nice to people he knew prestigious and unscrupulous. And finally there <sup>817</sup> (\*) **A. Beilinson** and **J. Bernstein** (I just do here <sup>812</sup> (\*) See note "The Perversity" n ° 76,

<sup>813</sup> (\*\*) This is a "good deal" who seems at the same time a very bad deal; and this even (and especially...) in the case where everything happens wishes for the person, wasting precious gifts and a creative force to play gangsters.

<sup>814</sup> (\*\*\*) See Note of the same name on March 22, n ° 173.

<sup>815</sup> (\*) A careful reader may be surprising not to find in this "parade of actors" (in the scam-mystification around the work of Zoghman Mebkhout) the name of Kawai, co-author with Kashiwara article repeatedly cited, including by. 4 pillages

shamelessly Chapter III of the thesis Mebkhout. (See the note about "The five photos (crystals and 3-modules" n ° 171

(Ix), in particular page 1005) Mebkhout insists that you can not put Kawai in "the same boat" with Kashiwara (be merely follow with his eyes closed. . . ). He described me as a guy just dropped and I felt he took almost affection - it is in fact his "good Japanese," and it does not matter to him that I touch! That is why also, no doubt, he refrained from writing it (as he had written to Hotta, another teammate Kashiwara), to report to him Scams in his article with Kashiwara and thereby put an obligation to show solidarity with its explicit teammate and boss.

816 (\*\*) This unusual printing had already imposed on me last year in the note "The Symposium" (n ° 75 Ⓒ)(which we guess...) given an atmosphere of racketeering as we would have said we dream, or that we are witnessing "a film about the reign of the Mafia in

shallows of some distant megalopolis. . . . "This impression was with me again, step by step, throughout the this peregrination through the misadventures of the unknown wave service. . .

817 (\*) (May 25) This "finally" proved premature - other gang members were reported to my attention since. On this subject 820

### 18.5. THE FOUR OPERATIONS (a corpse)

better acquainted), moved delicately by invisible strings. . .

And I wait without impatience and without illusions, what other Symposia Pervert the future holds with acquiescence without reserves of the whole Congregation for the greater glory of "Science" and for "the honor of the human spirit."

#### c3. Roots and loneliness

Rating 171 3

(April 18) On completion of this fourth day at follow step by step the misadventures of my Zoghman friend, I understand better than last year attitudes and dispositions, to me in particular, which had seemed strange even last year. In short, with his work that he felt the scope, he thought enter "a big family", just one of the deceased person whose master never spoke, this is true, and yet this even without talking about. And now he found himself in a world of sharks with polished tunes even affable, and the merciless teeth - a snap stripped of what he brought, the fruit of eight years of solitary work; after which he made clear that had seen enough: a and unwelcome intruders. There are not many in his place, which would have been traumatized. I do not know if it is open to a living soul on his troubles, if only by bitter allusions, and so vague that they are mine still testify against him as an embittered, associated it a little at the edges.

Though I will not be named, though I did figure of "Father" of this world without scruple neighborhood, and there was really no reason to trust me that. Our first meeting it is true, in 1980, while still a thousand miles from suspecting what was coming, had laid the foundation for confidence, and I did feel as towards and against all this foundation then was preserved until today. Basically, he knew, while "Father" of sharks as I am, that I was not going to like them. But there was a **grudge**, for sure, and she liked to take the air of mistrust would have desired visceral, and p. 1080 which nevertheless (and the least I felt) was "pressed".

It's easy to "fight" for what is believed to be its rights, when you're part of a group so small it may be, with which one feels in unison. But one who is alone against all, the excluded, unwelcome abroad, it is as a private tree from its soil. The force that is in him is of no help to him, it becomes bitterness who turns against itself, like to chime with the world, which rejects.

When I held the hands that book devoted exhumation patterns along the ENTER- surely the worker who made the show, this book signed by four authors among the brightest of brilliant generation (I helped train) - when I finally made aware by the greatest chance (because nobody until then had nothing noted in particular that was worth inform me of...) - to this time I knew for the first time in thirty-six years that I had become acquainted with the world of mathematicians, **I was alone against all**. Many things that had happened over the eight recent years, suddenly assembled and took all their meaning. It's a strange feeling when suddenly we rediscover this solitude- there. I have had to catch my breath on that day, and throughout the following weeks, taking cognizance every day of the whole dimension of the Burial - a Burial to the measurement of the work.

But that has nothing in common with Zoghman, "left behind" by his people even before he vraitment could take root. To me, the fate had smiled. With seniors who welcomed me (and no matter basically they are dead or retired and dealing may not math for ages) -

Note b. p. (\*) Page 962, in the note "The day of glory" (n ° 171 (iv)).

(30 May) Latest news: yet another member, R. Remmert, has been identified. See part (c) of this Note ("Fading memories - or the New History").

821

thanks to the fraternal welcome found in my younger years, I have, myself, "take root" in the ground that I myself selected. These roots have plunged and pushed, and over the years they have become deeper and powerful. These roots are then firmly planted in soil that is not that of "consensus" nor no way - deeper perhaps than any of those who find satisfaction in making modes and monitor 818 (\*).

I may, in short, of being "alone against all" - say what I have to say, and go my way.

p. 1081

(25 May) 819 (\*) It does not take much imagination to understand the frustration of Mebkhout that suddenly feels "swept" 820 (\*\*) like a wisp of straw, once the strength of its main result is recognized. He writes to me (in a letter of April 24, after a recent visit home): "I put eight years to climb the results used in the proof-Kazhdan Lusztig. They put a week at the show. "A modesty restrained him, again, to go after what he really felt, surely, and I take on I here add the "unspeakable": and once the thing done, "they" are strutted proudly together with the tool

p. 1082

another had brand new shapes in solitude, making clear to the workers that we had enough seen. . .

The thing is such a huge point, however, that instantly Zoghman do not yet believe, quite the testimony of his faculties healthy - just as I had myself not believe the testimony of my own, May 2 last year, taking knowledge of the Proceedings of the Symposium of Luminy 821 (\*). By taking Knowledge of these Acts in January last year, three years after the "General Rehearsal" Kazhdan-Lusztig that Zoghman eventually realize finally somehow what really happened.

818 (\*) If I never bothered to follow me or do fashion, either in mathematics or anywhere else, I know that this is a events just strong roots that I had the chance to develop in my childhood. Having had

upfront strong roots in myself, my energy mobilized in large investments is not dispersed by cravings compensation such cravings to set the tone, or to be and appear consistent with the "tone" of rigor.

I speak concretely about my childhood and about these "roots" (without that word, I think) in the note "The innocent cence (nuptials of yin and yang)" (n ° 107).

819 (\*) The two pages are from what was first anticipated as a mark of b. p. in note "... and the windfall" (n ° 171 (iii)). I had some hesitation where to insert them, and am finally decided to include in this note "Roots and loneliness. "This is the only note in" The Apotheosis "Indeed, I@ tried, from my own experience, to understand as well somehow how Zoghman himself experienced the events and situations that I got columnist.

820 (\*\*) The term "swept" is taken from a letter from Mebkhout (from two days before the one quoted in the main text), which I reproduce here the relevant passage:

"It is true that the constructability theorem [Kashiwara]... I was able to trigger me. Besides from this time someone like Deligne would have found in a wink all my results including the theorem of good God in all its forms, with demonstrations in four shots spoon as you say. This is why that all this was swept away in a few days. "

I think Mebkhout clarified there, exactly, the "reasoning" of a tacit Deligne, appropriating the fruit of labors of others because it **could have** (and **should have** ) find himself (with his means, luggage and all) "by four shots spoon. "The only problem with this reasoning there (that very often it is tempting to do in similar situations), it is that **the whole thing was to think about it** - and that@Mebkhout and not Deligne nor anyone else who has "thought" indeed. Creation

is not about the **technology** , which when finally seen something that nobody had been able to see, "sweeps" a situation in less time than it takes to write. Creation is not in the "scan", but in **the act of seeing** that person has been able to see; to see with his own eyes, not "follow" person. And this is part of probity in the exercise of the profession mathematician, that to distinguish between one and the other - between the act of creation, and the location of a crank which rotates round.

821 (\*) See about this Symposium (June 1981) note "Iniquity - or meaning of a return" or "Days of Glory" (n ° s 75, 171 (iv)). Actually, writing, during the first week of May last year, the "Procession VII: The Symposium - or bundles Mebkhout and perversity "(n ° s 75-80) has not been sufficient to overcome this inertia almost insurmountable "to believe the evidence of my faculties healthy" in a situation where it is strictly one to do use. Only five months later, finally seeing me face the reality "in the flesh" so to speak, in the person of my friend Pierre (Deligne) came to see me in my retirement, a secret and tenacious incredulity eventually vanish. On this subject the note "The accomplishment - or the moment of truth" (n ° 163), especially pages 782 to 784.

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## 18.5. THE FOUR OPERATIONS (a corpse)

The shock was terrible, I grew to understand - Zoghman at the time, he believed he was going to let them skin. It@solid, fortunately, a man - Zoghman is still alive, even today, and he even married meantime and became father of a child ... But I believe that even at that time again, when he held the hands of these "acts", he still could not believe it completely. Something had

"block". If it is, there still does not believe totally in the moment even as I write. I have to say that already in terms simply "rational" or "objective", the thing is so incredible point to such a huge point that until today **nobody** but me (except maybe him, and yet...) has dared to believe his eyes and see, while it is bigger than a cathedral!

But for one who is hit head-on by injustice, cynical and **free** the hands of his senior adminis- res, filled with all - surely this thing is those that can never believe everything-of-fact, those "**Beyond comprehension**". . . And these are also those who, thus, can devastate the life of a man. What gives them this destructive power is the obscure perception, hopelessly repressed and yet p. 1083 unexceptionable, of **the intent** to destroy, like that, for nothing, "**for fun**" - for the pleasure of a crush casual gesture which for you is the price, this same (if possible) that the substance and the salt of your life. This is perverse pleasure in malice "for nothing", that truly "beyond comprehension". . .

I believe that Zoghman has never really told anyone, either before the big bang, or after - if only by monosyllables, indecipherable to anyone but himself. The only episode Kazhdan-Lusztig already was too huge, too improbable for him to hope that anyone would believe it. The established consensus sweep as straws the most obvious facts, the most obvious, the most unexceptionable. And there it was something so painfully close to the point "on high" in his being, that the only risk to whom he would open reject the unwelcome message that his distress at "this beyond belief" is not allowed - this risk or probability that taking the size of **intolerable**, what we do expose any price - even to bursting on site, whether to die. . .

To me, two years ago, he had spoken well "monosyllabic". Maybe deep down he himself hoped that I would understand, these monosyllables, not only in their literal sense, but I also hear everything he dared not say in person (perhaps not even to himself ...). It was a crazy hope, certainly (in a situation where everything seemed raving mad!); I was miles away from anything imagine what I have since learned, of sure knowledge. It could not be otherwise, in the absence of meticulous and detailed information 822 (\*). And Zoghman, meanwhile, was a thousand miles too daring me to give that information. It was crazy, and that does not stop him from blame me. It was necessary that he wants someone, someone close enough, tangible in short, on which defer some at least from what had triggered in him by "what is beyond comprehension," and releasing so little either what gnawing.

#### c4. White card for looting - or High Works

Rating 171 4

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(June 2) It will be two months since I had the satisfaction to the "end point" in the p. 1084 Burial, with the final note "De Profundis" (April 7th) - and it©been two months since I worked hard to put "the last hand" to the last part of the Burial! This is the reissue, almost of what happened last year around the same time - while I was not finishing 822 (\*) (1 June) It would be more accurate to say he "could be otherwise" in the state of opening and limited presence that is mine, except on very rare occasions. I believe, however, that we all have an "ear, in the ear", perfectly able to hear the unsaid - but most often we take care to exclude from the field of conscious attention the messages caught by that ear. . .

823

finalize what was to be the first part of the Burial. It was, as now, the "last minute" that dragged on - even to the point that I forgot to drink and eat it and most of all, to sleep. It went on like that until the moment my body broke the ball rolling. It was exactly a year ago (within a few days), and I had to let go, for more than three months, fully busy to get out of an acute state of exhaustion 823 (\*). But this time I© suspicious, and I© very careful do not resume the same path. I care about my skin. . .

This time again, it was the "investigation" that never stops bouncing. I expected a score of ten pages to break everything, which would have name "The four operations" and which would summarize, by "putting in order", the results of last year©gale survey. And it©going to be four months since the investigation resumed from more beautiful, the ten pages have become three hundred or so, and again it is not yet (quite) completed ! I do not dare to make predictions - it©the ninth month since the resumption of work at the end of September, that I am "about to finish"! I know it©really finished the day only when the last packet of notes will have been typed in the net, read again and corrected, and handed over to the duplication. (After that, the rest is no longer my job.) All I know is that I can not wait to be there, as I look forward to seeing the end of a

long and grueling illness; and that I have to go all the way, the best I can do, without letting myself jostling with imaginary deadlines. I will stop to breathe only once at the end, as for all that was to be seen and **now** says, has been seen and said.

It is this damn "Apotheosis" that will have given me the most harm - I can not say why. These "four operations" are the only part of Harvests and Seeds that came together, in bits and pieces and struggling - when in principle it had to be cooked at all, a simple "putting in order" yes; nothing that engage or challenge my person so

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neuralgic, so as to mobilize resistance forces, a

p. 1085

"**Friction**". And yet God knows if there was friction, and with Apotheosis more than for all rest ! Where is he from ?

Already with "The maneuvers" it was laborious. This is where it began to stretch to infinity. It<sup>o</sup>ver by making eighty pages tightly packed just for that operation - and now, a month later, the Apotheosis has come to double. And yet, except perhaps a few pages (a little <sup>o</sup>ery "on the edges ..." in "The maneuvers" (where I enter, perhaps, more than it would have been indispensable in the stringy details of some "scam" not possible. . . ) - apart from this detailed "work on parts" and a bit of trouble probably for a reader who is not "in the know", I do not feel that these hundred-page packets that I ended up aligning there are superfluous, or even hair in four. What held me spellbound, it was precisely the abundance of **new substance** and unexpectedly poured over me, and that I absolutely had to fit, whether I like it or not - including same, yes, mathematical substance! At times I felt overwhelmed, so much many things at once I had to put black on white dare dare - things hot, see hot, and yet we are obliged to deal with them one after the other. . .

Such wealth, however, is in itself a powerful stimulation in the work, it is by no means of nature to arouse "friction", on the contrary. This friction, it is sure, does not come from the substance by itself, but from the strength of my egotistical investment in the work undertaken. Something that may seem paradoxical, it is my impatience even to "finish", to "throw on the carpet" what I have to say, about such and such things that are going on right now and that concern me and touch me closely - it<sup>o</sup>this impatience (I believe) that creates friction, the dispersion of energy. Friction is a sign of a division, forces pulling in opposite directions, each exasperating the resistance opposed by the other:

823 (\*) See note for the episode "The incident - or the body and mind" (n<sup>o</sup> 98).

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### 18.5. THE FOUR OPERATIONS (on a body)

in a hurry "to finish", to "let go" the piece since I polish it - and there is the requirement to go until end of what makes me glimpse the present moment, not to be satisfied with more or less, not to let me to shake things up, or to let me lock myself into a "program" to complete, in a "schedule" fixed in advance. I know although from the moment I exclude the unexpected, this hindrance to go around in circles, my work loses its quality and its meaning. he

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becomes "paper scraper". I became very sensitive over the years, this "small difference" p. 1086 who looks like nothing, and who is everything. It still happens, rarely, that such a turn begins, in moments of great heaviness - but never for long. When it takes this way, the kid he sends all wander - it<sup>o</sup>not even worth trying to keep going. The desire of the work itself, that **desire** is anything but the craving to accumulate pages or to place an endpoint - desire and sudden desire are gone, and you found stupidly to blacken paper. It is really more worthwhile then - it remains for me to correct the shot, and right now!

There is always some **impatience** in the work (an old friend of mine...), Who constantly pull me forward. It seems to me that this is not the same as the one that weighed heavily on me, since I have struggled with these "Four Operations". The other impatience is not a weight that weighs, but a pulling force. It is the sign of an appetite, not that of weariness or fatigue, or of satiety. It is not impatience to accumulate, or to finish, to "close" a program, but that to know the unknown before me, about to surrender. It is the impatience of the naked child, alone in front of the infinite sea, to dive into it to know it. . . 824 (\*)

But it is time to return to the story of the misadventures of my friend Zoghman, in this note intended as last end of the Apotheosis. As I said before, this story, Zoghman himself only gives it to me by snatches scattered, here and there, random letters, phone calls, meetings, surely, the progress of reflection and the writing of the Burial was felt, in the part, at least, devoted to the vicissitudes of my friend. I feel better now the meaning of this reluctance, while any attachment to a role of "victim" (that I had thought I detected last year) has fainted (assuming he was indeed present). There must have been also at home, at times, a certain saturation, expressing itself in an attitude like "does not throw



more, for pity! It should not have encouraged him, I was annoyed, I must say in a joke "the Japanese" here and "Kashiwara" there, that Zoghman had to sing for four years or five, and he had seen with them, it's true. But I knew that if he had seen, and if his work was thus delivered to the looting, so almost official: "Go good people, use galore, do not mind me especially...!", it was **not** because from some distant

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Japanese. It was **because of "his people"**: those of the "little family" <sup>825 (\*)</sup> - well <sub>p people. 1087</sub>

with us, and that he never named except to quote their works with all the respect due to their high reputation.

I did not want to hear about it any more, Kashiwara and others! Obviously it was blocking, and Zoghman had then the wisdom and patience to give up, without losing interest in my work, and without ceasing to bring here and there a discreet and effective competition.

It was at his last visit to my home, beginning of April, that I finally got to know him "Japanese package". It was a bit of my body, first of all. I thought I was going to piss off in inextricable ultratechnical stories and illegible papers (and in Japanese still, if it is ...), that anyway I will never read - and no! It was simple as hello - a little "pick-up story" pockets "in Parisian subways (or rather, from Tokyo) .Fun fun, to put it mildly (at least as long as it is the other who is mowing his wallet. . . ).

824 (\*) This is the picture already appeared in the note "The child and the sea - or faith and doubt" (No. 103 ◊).

825 (\*) (16 June) Mebkhout wishes to emphasize in this regard, it has entirely ceased to identify with the "little family" in question.  
825

And so the situation was unblocked between Zoghman and me, and I got snatches and pieces of his misadventures, by flashes, here and there. Episodes I had recorded a little in style "technical information sheet" were enhanced by reminiscences on the spot; the kind of things precisely those which seem to be banned forever from scientific texts, in their impassible "attention to you", and even letters between colleagues - you would not want to! He even had to shake me well, in "The Four operations ", so as not to fall back precisely in this style, the style" conclusions of investigation "(even," leaf "These scraps delivered by Zoghman will have helped me get out of it, and keep in touch with a living substance.

I knew I was going back to Apotheosis the same day that Zoghman was leaving my home, just to make a subnote or two more, as long as what he told me was hot again. That gave the notes (or sub-notes, I do not know for sure anymore. . . ) "Hatching of a vision - or the intruder", "The Mafia" (which I subdivided by the continued in seven parts, each with a name), and "Roots and solitude". I sent him the whole thing, for him to give me his comments before I give it to the keystroke. I felt like I was expressing myself a little bit in his name, and I wanted to be sure that everything I reported, according to what he had told me, had his unreserved approval. He sent me his detailed comments by return (letter of April 22nd and 24th).

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In these comments there are a lot of these "snatches", putting a living flesh on a framework of facts that p. 1088

seems a little skeletal at times, in my notes.

That's how I also knew that Zoghman had been there, this memorable April 22, 1980 seminary Goulaouic-Schwartz. This is the day when Kashiwara announced as his theorem the theorem of God, he had learned from the mouth of Mebkhout few months earlier, the Symposium des Houches <sup>826 (\*)</sup>! It's so big, and with Mebkhout in the room again, that it may seem incredible. Mebkhout did not exploded on the field (I wonder how he did it ...). He waited politely for the end of the presentation "for publicly protest these methods, reminding him of the conference Houches and issue <sup>827 (\*\*)</sup>.

Goulaouic asked me to settle my stories in private. The room suddenly emptied in a few seconds.

So here is one of the "snatches", delivered by this terse description. I then had some details at phone. The incident deserves attention. It says a lot about the state of morals in the mathematical world, in the 80s. There is the mentality, not such "caid" with long teeth, extreme symptom of the decomposition of traditional values in the scientific world, or even of the "establishment" of people in sight and well in all respects, in which plays the class reflex in favor of a "theirs". Here it is any room that is empty in a wink - nobody suddenly <sup>828 (\*\*\*)</sup>! Arrange between you - we do not want to know anything. . .

I wonder what could have happened in the minds of Goulaouic and other peaceful listeners in this seminar, where spoke a distinguished foreign lecturer (on a theme of which none of them, I believe, was too familiar). This incident, after all, was sobering. I doubt

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besides that no

p. 1089

of them had taken this trouble, and supposes rather that all of them agreed by mutual agreement to forget the

incident. But finally, if we take the trouble to think about it instead of running away, there was still **one** things that was clear, in this dark history. The tone and terms of Mebkhout (someone else they knew, for having rubbed shoulders with him in seminars at least), did not

826 (\*) About the Symposium Les Houches and the episode of Goulaouic-Schwartz seminar, see note "La Mafia" (No. 171 °), part (b) "First trouble - or the overseas bosses".

827 (\*\*) This is the question posed by Kashiwara at the end of the presentation of the Les Houches Mebkhout Symposium in September 1979.

See in this regard the note quoted in the note of b. from p. who is before.

828 (\*\*\*) this evocation irresistibly raises in my mind the association of ideas with any similar situation that I had lived three years ago, at the end of a Bourbaki seminar where I had been given ten minutes to talk about a some law scoundrel striking strangers. On this subject the "My farewell, or: foreigners", n ° 24.

826

### 18.5. THE FOUR OPERATIONS (on a body)

little doubt about this, there had to be a **con man** in history - or Mebkhout or Kashiwara. It is possible, of course, that, in their heart of hearts, they have already decided in advance: Mebkhut is fabulous, how could one imagine the distinguished visitor plundering the anonymous listener! That would mean that vis-à-vis an unknown, the famous man, whatever he does, is above suspicion: the **white card looting**, given to man awareness against him who is without recourse. What he will have to say will not be heard: "arrange between you!"

Or, they are buried in a state of doubt: how to know who says the true and the false? (And especially, again, if we stop talking!) It is true that the brutal base of a Kashiwara, publicly looting a unknown wave in the presence of the person concerned, seems barely credible. But it would be a more incredible thing still after all, a vague stranger (whom they all know, and who did not come to their attention again by rogue tricks or by his base. . . ) publicly dares to accuse a Kashiwara of gross plagiarism, what he has to say is pure fabrication. . . And supposing that what he says is perhaps well founded, to send it on the roses with an "arrange yourself between you!", this is again the *carte blanche* for looting. It is like shouting at someone who gets robbed in the street by thugs in a tuxedo and shouts "at Thief! "- Arrange between you! "

It also seems that this is how it has been for a long time, in the neighborhoods of New York and other major American cities, where no one wants to get in trouble with the mafia that makes it law. That is the way it is going nowadays (I can not say since when), in the world mathematics and in what passes for the "uptown" as the Goulaouic-Schwartz Seminar 829 (\*), or among all those prestigious people who "make" cohomology of algebraic varieties.

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In rational terms and taken literally, this "you arrange you" frieze debility, in a p. 1090 situation where it is clear in any case that one of the two parties must be in bad faith. At the psychological level, this stupid formula reflects a **resignation** responsibilities, a situation experienced as "troublesome". It is also the deliberate ignorance of this obvious fact: the question of respect for basic rules the ethics of the profession of mathematician is not a purely "private" affair, to be settled between who arrogates to despise them, and whoever pays them. This is a **public matter**, a matter that concerns **every** mathematician.

It is thanks to the general indifference, the panic of each to assume his personal responsibility, in the scientific world can flourish with a gangster mentality and shameless than that of the Pervers Symposium. The panic of some and the impudence of others are as wrong and the place of the **same corruption**. Those who ran away and stopped their ears, a certain April 22, 1980, contributed to the Apotheosis of the memorable Symposium the year after, as well as the men who have assembled the grandiose mystification from scratch and have strutted proudly there. (June 3) It was during Mebkhut's last visit to my home, too, that I had some edifying details about him. about some of the participants in this brilliant symposium, and the "new style" that flourishes among and others, who better. I had the opportunity to leaf through the record of work, in the second volume of the Acts, where there are articles by Verdier and Brylinski-Malgrange, and to take a look at the Laumon thesis (with a more informed and less distracted eye than the day I first received it). This thesis is in fact a work in collaboration with N. Katz. I give some comments about the "new style" followed in these works, in the long note from b to the note "The day of Glory" (God knows she deserved that name. . . ), page 962. In this note I refer, for further details, to this note.

829 (\*) I am pleased to mention here that Laurent Schwartz was not in the room the day of memorable incident to "his" seminar. I do not know if he was informed later.

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(not yet written at this time). Promised thing, thing due!

Mebkhout told me how he had the honor and the benefit of talking to N. Katz twice about his ideas on duality and on the links between continuous coefficients and discrete coefficients. The first time was at Symposium of p-Adic Analysis in Rennes, in July 1978. He then explained

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"in a small committee" his theorem of

p. 1091

global duality for 3-Modules, on a complex analytic space - the theorem that covers the duality of Serre and the Poincaré <sup>830</sup> (\*). There was Katz and Illusie, the very ones who have already been talked about more than one times in the Burial. Illusie, him, kind and nice as it is his habit, thought it was really very nice - something like that <sup>831</sup> (\*\*). As for Katz, who I imagine was talking about 3-Modules for the first time in his life (at a time when it was far from being great fashion, as after the memorable Colloquium), he just said dryly "It's known that!", To turn his heels as dry. Of moment it was a vague Mr. Nobody who spoke to him, to him N. Katz (who that same year was going to give a speech to thousands of distinguished colleagues, in honor of the new Fields laureate Pierre Deligne. . . ), it could only be "known".

The second time it was shortly after the Symposium September 1979 <sup>832</sup> Houches (\*\*\*)). Katz was then at IHES. Given his notorious competence in p-adic differential systems, which Mebkhout felt it had something to do with the theorem of the good God he had just spoken to Houches, Mebkhout is went expressly to IHES to bring him his article at Les Houches, and to talk about his ideas and results. After the welcome received in Rennes, we can say that he had more ideas, not get tired! There is always that it was a bit of the same scenario. Katz still received from very high this vague unknown, who allowed himself to come relaunch it a second time, and without announcing again if it is. When you are an important man, sometimes we do not know how to protect ourselves from the nuisances. . .

It will have been enough, a year later, that these same ideas, long worn and matured in solitude by a unknown wave, be trumpeted everywhere as the last of the finds of a Deligne (or a Kashiwara, we did not know too much ...), in the wake of such a brilliant Conference that Katz unfortunately could not honor of his presence, so that suddenly they take

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for the big man and the importance and weight.

p. 1092

It was surely Laumon who had to explain to him the ins and outs - one of the brightest disciples Deligne. This same Laumon also knew, too, and first hand, the origin of these ideas, for being informed by the unknown wave in person. But the disciple is honored to follow in the footsteps of the Master, and he had shown quite clearly, and unequivocally, what conduct he it was appropriate to adopt with regard to the one devoted to silence and darkness. To Deligne and Verdier the honors of the limelight, and Brylinski, Katz and Laumon, come at the right time to have their share! To them the music and the flons-flons, and the ovations of a grateful crowd, hastily rushing to celebrate these High Works, in the hands of his New Masters.

### **Epilogue Beyond the Grave - or Racking**

Note 171

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(June 14) Until a month ago, it seemed to me that the spirit of the Burial was limited

p. 1093

to what I sometimes call "the beautiful world" or the "big world" mathematical, and more particularly, the <sup>830</sup> (\*) It is a question of this theorem in the two notes and "The Work..." "Three milestones - or innocence" (n ° 171 (ii), (x)). <sup>831</sup> (\*\*) It was also where a "kindness" gratuitous. While the style of reaction was different from one to the other (in "yin" at Illusie, in "yang" at Katz), the bottom was the same: as long as it comes from Mr. Nobody, it goes into a ear to go out by the other! On this subject the note "Spoofing" (n ° 85 ☹) including my observations about Illusie on page 351.

<sup>832</sup> (\*\*\*) About Symposium Les Houches and fraud Kashiwara seminary Goulaouic-Schwartz, see note "The Mafia "(No. 171 °), part (b)," Getting into trouble - or the guys overseas peaceful. "

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### 18.5. THE FOUR OPERATIONS (on a body)

backgrounds of this world that I used to haunt and that I myself was part of. I did not perceive at the USTL (University of Sciences and Techniques of Languedoc, Montpellier), which has been for twelve years my institution of attachment, of signs of ostracism, or of an affectation of misery or discourtesy, even those of rudeness, along the lines of this Burial full swing for fifteen years <sup>833</sup> (\*).

A new fact has just burst into this peaceful picture, and drastically transform the said chart, and my own relationship to my home institution.

In accordance with inveterate mechanisms, I did not think at first to include in my testimony

"Crop and Seed" this recent incident, which at first glance seemed to me to be coming "like hair on soup. "It is against serious resistance that I have come to admit that it would be failing in the spirit of my testimony, than to ignore this episode. It is an episode all fresh yet, certainly, and one, of more, that I "cashed" hard enough - which gives additional strength to these "mechanisms inveterate" to which I have just alluded, but the very vivacity with which I have cashed, this time, the eloquent and unwelcome teachings of this incident, is also a sign that it touches me very closely - and this at the level of my professional activity and my links with the professional environment of which I am a part. he this is typically the kind of thing that Harvests and Seeds would like to witness deep, without a "reserved corner" to which I would forbid myself to touch, whether by a "discretion" misplaced vis-à-vis myself, or vis-à-vis anyone.

Moreover, in the more particular context of my reflection on the Burial, I feel as a matter of course that there are direct links between it, and the incident in question. It is possible that these links are not those of a simple cause-and-effect relationship: that some colleagues on the spot would eventually take note of the Burial, and would have concluded that they too could "give themselves". Even as he there would be such a link of cause and effect, it would affect, it seems to me, only an incidental, accidental aspect of the situation. A more essential aspect on the other hand, and which especially struck me, common to what happens in "the big world" of Science (with capital S), or in a modest provincial university, is some

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**degradation**, unprecedented perhaps in science and academia: degradation at p. 1094 relationships and basic forms of courtesy and respect for others, such as at the level of scientific ethics, itself indissolubly linked to the respect of others and of oneself. We can therefore consider the following pages as a contribution (among many other already provided all along the reflection on the Burial) to "table manners of a time," or time without end doubt, mathematical environment.

Rather than repeating a story more or less circumstantial events, I prefer to reproduce four **Documents** that describe them as well. It is about :

1. a "letter to my mathematics teacher colleagues in USTL", dated 28 May, where I informed of a certain situation and expressed the hope of a discussion in General Meeting;
  2. "response" Ms. Charles, head of the local mathematical building to USTL, in the form of a circular letter dated 30 May addressed to me by name, and in fact to all mathematics teachers;
  3. Resolution passed by the General Meeting of the EBU 5, held on June 6 on the Agenda: "Information and discussions about the office move of Professor Grothendieck"; and finally
  4. a "Letter to my work colleagues in former Mathematics building", dated the following day June 7
- 833 (\*) I am speaking particularly in the sense in Note 93 (page 396, paragraph 3).  
829

I abstained included among the documents my letter to Mrs. Charles May 21 (referred in the first document cited) and my letter to Mr. R. Cano, Provisional Administrator of USTL (he discussed in the same document and in document 4 or "Epilogue a misunderstanding"); these letters I seem to bring any new information element, compared to those contained in the documents reproduced below.

As one comment on Ms. Charles' letter ( "it is in fact very difficult to contact" - "the" here means my humble person, to whom the letter is supposed to be addressed), I said here that the letters Montpellier to put my home one day to happen to me, and for years I am away from my address that during my visits to the USTL.  
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## 18.5. THE FOUR OPERATIONS (a corpse)

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UNIVERSITY OF SCIENCE AND TECHNOLOGY OF LANGUEDOC  
p. 1095  
Institute of Mathematics  
GETTING AN A BAG IN MATHEMATICS BUILDING  
Letter to my mathematics teacher colleagues in USTL  
Alexander GROTHENDIECK  
Montpellier, 28.05.1985  
Dear Colleague,  
I was informed last week by Secretary of the EBU which I charged to go for a

Work being in my office on the fourth floor, that it had been emptied of all my stuff - something I could check today: there are only bare soil. I had not been informed that my office would be requisitioned without further ado, and so I had not been able to express my agreement the operation, let alone allow anyone to enter my office in my absence and to touch my business. I called the same day to Mr Lefranc, director of the EBU to inform him of the situation, which (it seemed) was the fact of an initiative of Mrs. Charles, something that seemed to confirm this phone call. I pointed out to Mr. Lefranc I was shocked by the process, there was no question that I give my consent to a transfer offices are doing in such brutal forms, and I expected that my affairs are put in their place as soon as possible. He assured me he would need. That same day yet, Tuesday, May 21, I wrote to Mrs. Charles to tell him that I considered the "Dump" unwanted my office for abuse of power, and felt as violence; than I expected to detailed explanations from him, and apologize unreservedly. What if otherwise, I would submit the matter to the Council of the University, which would decide whether such processes against a teacher at the USTL should be considered as accepted thing. Coming now to the USTL, I noticed that Mrs. Charles did not bother to answer my letter (which I also sent a copy to Messrs. Cano and Lefranc). Mr. Lefranc has not ruled useful to send me any explanation for the fact that my office is still empty my business, a week after he assured me that he would need to return to my office. Neither he nor Ms. Charles had seen fit to inform me where the cases that have been rounded up are. I knew by secretaries interposed that these cases would be stored in the office of one of them. Moreover, having been an opportunity to meet Mrs. Charles in the boardroom, it assured me she only followed the instructions of the Director of the EBU, Mr Lefranc, and invited me to speak to him for that matter, that does not concern. Until the situation unravels, Mr Nguiffo Boyom kindly share her office with me.

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I may be the one to find that there is something wrong - violence and contempt; it is p. 1096 Although I am the only one also pretended to and at the door without further ado. (If it is another besides me who thinks this is not the kind of atmosphere in which he wants to work with the USTL, it would make me really happy he made known to me. . . 834 (\*)). For my part, I consider it would not be a luxury there, following this "misunderstanding" (to use the charming euphemism of one of my colleagues), a meeting of the EBU, to the Director, M. Lefranc, and Mrs Charles, the opportunity to 834 (\*) It goes without saying that such a gesture has for me makes sense if it is understood that commits the signatory, which allows me to make state

publicly.

831

to explain their intentions and motivations, and teachers of the EBU, to say whether they consider these processes as normal (when applied to others...).

For twelve years I have to USTL, I have often had occasion to appreciate the provisions well-veillantes, dedication and efficiency of M. Lefranc whenever he was to serve - and I am grateful. This is all the more regret than I would withdraw my trust him, seeing that he is an instrument in the hands of others and allowed to establish an atmosphere of arbitrariness and contempt. Of his Now I pray to assume its responsibilities Director of the EBU, or to resign.

And I pray Mrs. Charles to dismiss "responsible local" EBU, functions which it has pleased him to abuse.

Pending your (or your) response

Alexandre GROTHENDIECK

PS As a temperament inclined to help, I was last year at the request of Mr. Lefranc given my agreement to exchange offices-with Mr. Lapacher, which (to me he was told shortly after) a then changed his plans. It goes without saying that gadfly agreement did not mean that I allowed the sacking of my office at that time or any other.

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18.5. THE FOUR OPERATIONS (a corpse)

UNIVERSITY OF SCIENCE &

TECHNIQUES

THE LANGUEDOC

MATHEMATICS

Thursday, May 30, 1985

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Mrs J. CHARLES "responsible local to the Institute of Mathematics"  
p. 1097

Monsieur A. GROTHENDIECK, Professor of Mathematics.

Dear Colleague,

1. Where to begin and where the stop "work" of "responsible local to the Institute of Mathematics"?

This "charge" is seized of requests for Mathematics teachers

- or to accommodate a new teacher (or researcher)

- either for staying outside a teacher (or researcher) already housed.

In this second case the requests are usually motivated for a work goal: consolidation members of the same group.

This "charge" while exploring opportunities first priority and with the director of that UER5 is officially the manager appointed by the President of the USTL to the premises of the building Mathematical Research. It then seeks with people concerned possible solutions; the modification occurs after agreement by all.

2. What has been done so in recent years:

- grouping members geometry group

- association of members of the mechanical group

3. Difficulties encountered in this "work":

- virtually every person contacted feels "owner" of his office

- it seems impossible to compel that whatsoever to "change" Office.

4. The last request received by me and the evaluation of research "solutions" to the problem:

- the request made by Mr. LAPSCHER professor: regroup at the same level Mr.

LAPSCHER, the office of his secretary, Mr. MICALI.

- the first proposed solution: exchange offices between third and fourth floor for the

"Applicants" are grouped on the fourth floor. Were particularly concerned by this exchange

Mr. and Mr. GROTHENDIECK THEROND. Mr. GROTHENDIECK contacted

Director of the EBU 5

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said it mattered little to him that THE LOCATION OF ITS p. 1098

OFFICE PROVIDED THAT HE HAS A. As against Mr. THEROND with one point agreed then refused any movement.

- the second proposed solution: I then asked Mr. LAPSCHER contact itself

even his colleagues to propose another solution; this was confirmed by the director of the

EBU 5. He made us aware of his actions: the "occupiers" of 5 offices agreed

to perform a switchover, the agreement of Mr. GROTHENDIECK result of his conversation with the director of the EBU 5.

- the completion of this second solution: after learning of the agreement director

EBU 5 gave the "green light" for the modification of proposed offices.

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Mr. LAPSCHER who told me about a key issue during the period when the move would be addressed but I did not finish him that

- no new key was probably available,

- it did not seem desirable to prolong this move could be done in a few hours with the participation of all concerned.

Mr. LAPSCHER me then informed that the office equipment Mr. GROTHENDIECK was transported in his future office; This had been done without being able to reach previously Mr. GROTHENDIECK.

It should be noted that Mr. GROTHENDIECK resides away from Montpellier and is currently in release position at the CNRS; it is actually very difficult to contact.

5. My impression of "responsible" on what would seem to be called a "conflict":

- I had the opportunity to clarify to Mr GROTHENDIECK that, acting on behalf of the EBU I 5 could not give myself to reply to his letter; so he had to ask a response to

Director of the EBU 5. Following this second letter to all I consider that I have to leave

"The obligation of discretion" that I had imposed.

- it would have seemed desirable at least to inform the persons concerned before their move equipment

- it would have seemed desirable also to make the move into a 1/2 day maximum.

- the proposed solution seemed valid, it did not affect the office occupancy rate of the persons involved.

I do not expect an answer.

Please accept, Excellency and dear colleague, the expression of my best regards.

NB A copy of this letter for information

- all Mathematics teachers who have received the letter from Mr GROTHENDIECK of 28/05/85.

- the director of the EBU 5 receiving more copy of the letter that was sent to me by Mr.

GROTHENDIECK the 05/21/85.

- the temporary administrator of the USTL, who had a copy of the letter of 05/21/85 and to which I attach a copy of the letter of 28/05/85.

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18.5. THE FOUR OPERATIONS (a corpse)

UNIVERSITY OF SCIENCE AND TECHNOLOGY

THE LANGUEDOC

Institute of Mathematics

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INSTITUTE OF MATHEMATICS

p. 1099

Minutes of the meeting on Thursday, 6th June 1985 in 18 hours.

Present: M. AUBERSON Ms CHARLES, MM. CIULLI, CONTOU Carrere, MM. CUER,

LIMA, DELOBEL, ROBERT, GROTHENDIECK, Hocquemiller, ESCAMILLA, Mie HU

BERT COULIN "M. Lefranc, M. LOUPIAS Ms. MEDEN Mr. MOLINO, MmePIERROT Mr PIN-UP

CHARD, Mr. SAINT PIERRE, Miss VOISIN

After discussion, the present (19) adopted by 16 yes and 3 abst. the following:

"Mathematics Teachers present their apologies to Mr. GROTHENDIECK to

about unacceptable conditions in which his affairs were displaced. They commit

to collectively ensure that such regrettable events do not recur. In particular,

should be clear that the key to an office can not be used by anyone without the explicit consent of the occupier. "

M. LEFRANC

director

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UNIVERSITY OF SCIENCE AND TECHNOLOGY OF LANGUEDOC

p. 1100

Institute of Mathematics

Epilogue of a "misunderstanding"

Letter to my work ex-colleagues (teachers and technical, 3rd cycle students) to the building

Mathematics

by Alexandre Grothendieck

. . . the 7.6.1985

Dear (e) Colleague, I write here epilogue to the case of making my office bag, mentioned in my letter

of May 28 This letter was addressed only to teachers of mathematics, then it concerns EGA-

LEMENT and alongside all those and all those who hold office in the mathematics building.

It©inadvertent and poor judgment that I had failed to address my letter also to

Technical staff and students of 3 ° cycle, judging (hastily) that this would give the incident a

extension did not return it. I sincerely apologize to the person (s), and this especially as

I received from several of them (supposedly uninformed...) of expressions of sympathy, which made me

touch. It was after this oversight also no doubt that the General Meeting of the EBU, spent yesterday

the incident was limited only to "members of the EBU 5".

Among many other things, this incident has made me learn that this is not the first of its kind that

occurs at the EBU 5 - it©only the first time that it is a "rank A teacher" is targeted. I

do not know if the pious resolution passed yesterday will prevent such incidents from recurring, in indifference

General (as before), vis-à-vis non-tenured teachers or students of 3 ° cycle particular. I take-

drai care to check with Ms. Mori and Mrs. Moure if they have received the instructions

by the director of the EBU, no longer any reason to entrust the key to the office or to anyone

make use for anyone, except with the express permission of its occupants.

My previous letter ended with the words "waiting for your (or your) response." In response to this expectation,

I received three expressions of sympathy and solidarity. They come to me from Louis Pinchard to

Pierre Molino and Christine Voisin. Also, I received a testimony to the same effect by Philippe

Delobel, student 3 ° cycle (as Christine Voisin) had an AED with me. It was at his initiative

some students of 3 ° cycle yesterday attended the General Meeting. To him, as to all those I

just mentioned, which made me (without ambiguity or dodges) expressed their solidarity, I am pleased to express here my appreciation and gratitude. This is a "hard" fruit of experiences like this that make recognize friends, when we are lucky to have them. . .

I received yet another letter responding to mine, from a clearly delighted colleague what coming and taking this opportunity to file kindly of me. It is the only echoed in the sense that I have collected. For all others, many total indifference of some and discomfort of others (where more than once I felt the unspoken fear of hurting view and thereby jeopardize his chances of promotion or a precarious situation). In all, among those who are upset to the point of bothering to attend at this General Meeting (convened in haste at the last minute, when it was scheduled for a week. . .) I especially felt the deliberate well stopped to drown a fish, to the tune of "everyone there nice, everyone is cute." It was finally turned down (after three quarter hour of palaver) the "ugly" all designated absent (coincidentally), Mr. Lapscher - who had taken (from what we had just imply)

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the initiative of the hand. There was no question of going up

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put into question, namely, the poor - not more than did anyone else, it goes without saying.

From the "leaders" involved in one way or another in the incident of the sack, I was

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#### 18.5. THE FOUR OPERATIONS (a corpse)

shocked by the brutality of shameless Lapscher by rudeness "for fun" of Mrs. Charles (Which covered the hand, once put a fait accompli, adding there insolence of his own) and the discourtesy of M. Cano, Provisional Administrator of USTL, dispensing with any response the letter in which I informed him of the situation and to bring to the Council prayed University. But more than while I was baffled and saddened by the ambiguous attitude of Mr. Lefranc, director of the EBU 5. Since Monday, May 20 (where I had informed of the situation I had to discover and my feelings about it) until yesterday he had not seen fit neither to inform me about what had happened, nor to dissociate unequivocal act of robbery of a Lapscher or coarseness of Mrs. Charles. By his possible, from beginning to end, to maintain the fiction of the unfortunate "misunderstanding", he managed to give innocuous or even respectable manners, to behaviors that, for my part, I feel as intolerable not to hurt anyone, surely, he chose to spare (a lot) and goats (some) cabbage.

I have also taken note, among other signs, the silence of many of those I had grown count among my friends (including three who were my students); of ostentative indifferent of such, of the embarrassment of another, and honey jubilation another again. And the silence of a Micali (co-recipient@hand, and had ample opportunity to convince the last few years, disadvantages in attracting the bad graces of Mr. and Mrs. Charles. . .), And complacency Miss Brown, taking orders from a Lapscher to play locksmiths-movers mercenaries (without a word of regret, once the nature of the operation could not be sure).

On the bottom of all this and finding yesterday that, for twelve years, was my desk, turned this Once in battle - my business (more furniture) réentassées crash (good fifteen days after a hand - lightning. . .) - I have not the heart to present to redevelop it again. It@quite unlikely, do we ensure me that the same incident happening again vis-a-vis me, and I can also take the lead, taking before me the second key, until now entrusted to Ms. Mori and Moure. But in extent practically possible, particularly for the duration of my detachment CNRS, I prefer to give now to the use of an office in the USTL and abandon the place without a fight to Lapscher, to Charles and others.

If I can avoid it, I will not repeat a teaching activity to USTL. I have spent, for sure, as a stranger - one whose homeland is elsewhere - both my approach to mathematics, as by that teaching or my lifestyle. What the university microcosm had to teach me, I think I have learned, with the last "shutter" the lessons of this incident, which has just closed himself in satisfaction. It@likely that this meeting of the EBU 5 that I just attended either last, that the letter also be the last opportunity I have to write (or write yourself). And this time, I do not expect an answer.

Alexandre Grothendieck