Supplementary Information

This Supplementary Information accompanies the "Stochastic Interpolation Model of the Medial Superior Olive Neural Circuit" paper by Pavel Sanda and Petr Marsalek in the *Brain Research* journal.

This supplement contains an extended comparison of the circuit features for different jitter values, a comparison of asymptotic behavior of our model with asymptotic behavior when the Meddis auditory periphery model is used, and finally a detailed list of circuit parameters.

1 Comparison of Two Magnitudes of Jitter

The figures in this section offer a side-to-side comparison of jitter effects for $T_J = 1$ ms and $T_J = 5$ ms in model simulations.

- Figure 1.1 shows the ITD (interaural time difference) curve for basic parameters, inhibition and excitation cases.
- Figure 1.2 shows the dependency of the ITD curve on input frequency f_{in}.
- Figure 1.3 shows the dependency of the ITD curve on coincidence window width w_{CD}.

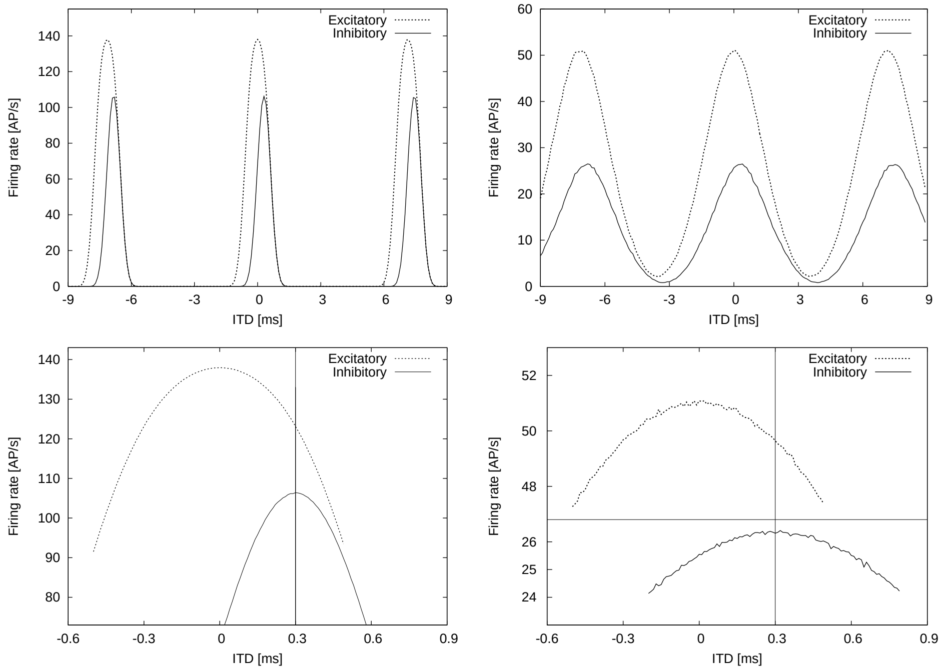

Figure 1.1: **ITD curves for the basic set of parameters.** Left-hand panels $T_J = 1$ ms, right-hand panels $T_J = 5$ ms. The bottom panels show detailed view of the ITD curve around $D_{ITD} = 0$ ms. We can see that the curve is periodic and its peak is shifted due to inhibition. The right bottom panel shows a detailed view of the peaks (vertically shifted), note that the interval from 27 to 47 AP/s on the y-axis has been deleted for clarity.

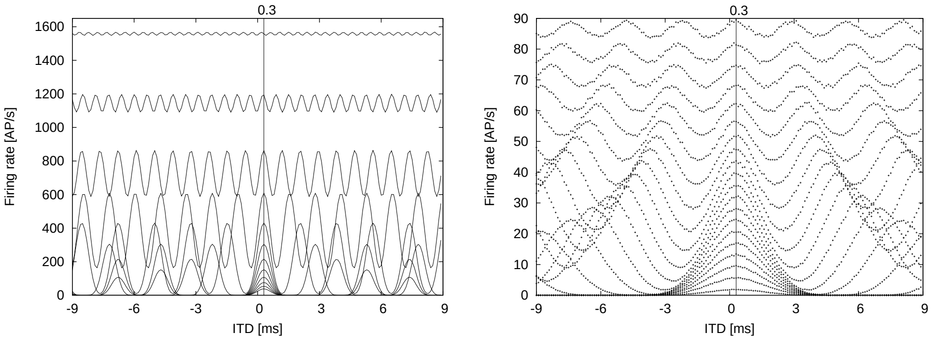

Figure 1.2: **ITD response curves for different input frequencies f_{in}.** Left-hand panel $T_J = 1$ ms, right-hand panel $T_J = 5$ ms. Left-hand panel: For increasing sound frequencies with step $\sqrt{2}$ (..., 566, 800, 1131, 1600, 2262 Hz), the maximum firing rate gets higher. Right-hand panel: Because of the higher jitter magnitude, coincidences occur for all ITD values and this deteriorates the circuit function faster compared to $T_J = 1$ ms by the fact that around higher sound frequencies (~ 400 Hz) the periodic curves already become more flat, letting only small output dynamic range for azimuth encoding.

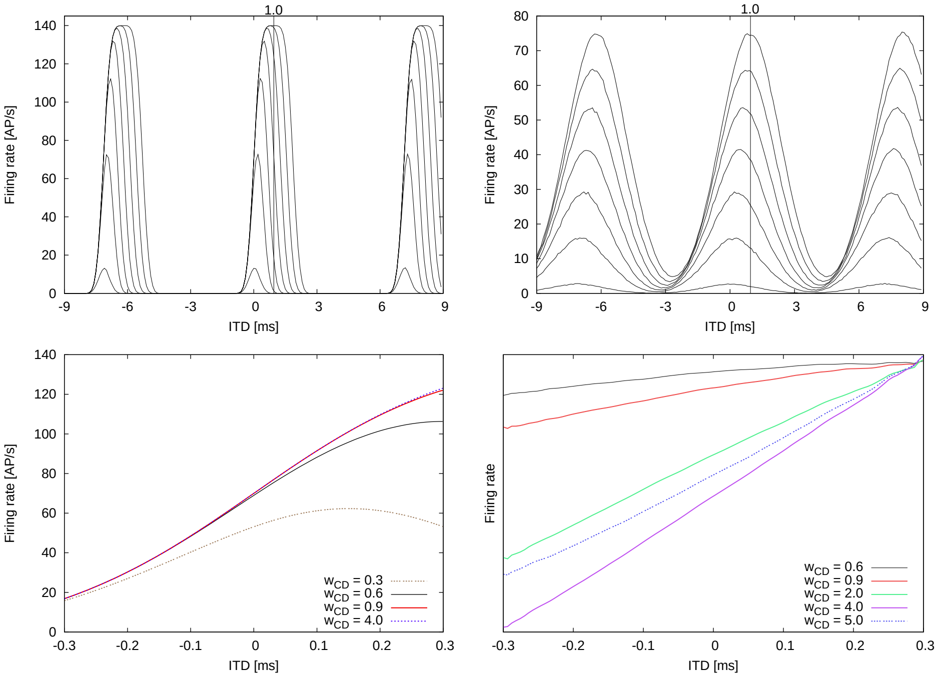

Figure 1.3: **Dependence of the ITD peak shift on the coincidence detection window width w_{CD}.** Left panels $T_J = 1$ ms, right panels $T_J = 5$ ms. Left top: Increasing window width causes peak shift, from about $w_{CD} = 0.6$ ms the shape of the ITD function in physiologically relevant range does not change. Peaks do not exceed 140 Hz due to the input frequency $f_{in} = 140$ Hz in the basic set of parameters. Right top: Larger jitter causes lower output dynamic range and wider curves. Both top figures are generated for w_{CD} ranging from 60 μs to 2 ms (with step 300 μs). Bottom figures show details in the physiologically relevant range. Left bottom: Wider coincidence detection window shifts peaks and increases total output range. Right bottom: All the curves were shifted vertically to make their firing rates pass through the same point at $ITD = 300\,\mu$s, so their slopes can be easily compared. It can be seen that for basic set $w_{CD} = 0.6$ ms has far smaller steepness and output range. With wider coincidence detection windows, the slope starts to decrease again, see $w_{CD} = 5$ ms.

2 Asymptotic Times of Observer with Meddis Model

Below is a comparison of asymptotic times of azimuth estimate by the ideal observer module when a more realistic model of auditory periphery, published in Meddis (2006), is used. The output of auditory nerve (AN) fibers of that model is used as an input to our circuit. There is, however, one missing stage of signal relaying by the bushy-cells neurons, which provide the input to the binaural MSO neurons. This stage might improve the phase-locking of spikes, thanks to the integration inputs from more fibers in parallel and change the considerations about the number of binaural neurons needed in order to reproduce psychophysical experiments. Since we are not aware of any published complex computational model of auditory pathway (up to the stage of bushy cells) used in this context, the findings about the precision performance of our circuit to the real sounds stimuli may vary depending on the particular peripheral processing. This issue merits further investigation in the future.

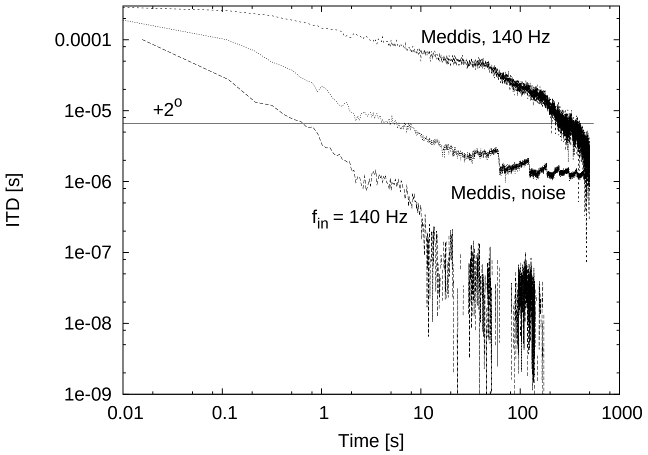

Figure 2.1: **Time required to reach a reliable estimate of azimuth T_A by a single binaural neuron.** x-axis shows time, y-axis the ITD. Note the logarithmic scales used. The bottom line shows convergence for the basic set of parameters when regular spiking input (140 Hz) is used. The middle line corresponds to the model when the white noise waveform input processed through the Meddis model and its output is subsequently used in our circuit. The top line, showing the slowest convergence, corresponds to the 140 Hz pure tone stimuli given to the Meddis model and subsequently used in our circuit. In both cases we use two independent AN fibers with characteristic frequency CF=140 Hz. $T_J = 1$ ms in all three cases. The flat line shows precision obtained in psychoacoustical experiments. At the point in time T_A when the mean value crosses the threshold, enough information has been obtained for our model to be able to report the azimuth with average precision corresponding to that of human listeners ([-2°; 2°]).

3 Summary for Circuit Details and Parameters

The following list contains the default values used in simulations unless stated otherwise in the text.

- Spike generation
  - Regular generation
    Input frequency $f_{in} = 140$ Hz.
    Total time $T = 500$ s.
  - Meddis model of periphery. Model is identical to Meddis (2006).
    Two independent AN fibers.
    Input sound waveforms are pure tones, having the same CF of AN fiber as the input tone frequency.
    Total time $T = 500$ s.

- Jitter generation

  Jitter parameter $T_J = 1$ ms.

  Actual jitter added to the spike $D = T_J(B(2,4) - 0.5)$ ($B(2,4)$ is the beta distribution with parameters 2 and 4).

- Coincidence detector

  Coincidence detection window $w_{CD} = 600$ μs.

  In order to show how the coincidence detection is implemented in the simulation, we list the relevant part of the source code as edited pseudocode similar to the C language conventions.

  - Excitatory case

    Output spike is generated whenever two input excitatory spikes occur within the coincidence window.

```
last_excit=-1;          //time of the last excitatory spike
                        //-1 when no previous spike is available

Excitatory_CD(spike){                   //coincidence detection function
                                        //input are excitatory spikes

        if (last_excit==-1 && spike.type==excitatory) {last_excit=spike.time; return;}

        if (spike.type == excitatory){
                if (spike.time-last_excit < w_CD) {
                        CD_Fire_Output_Spike(spike.time);
                        last_excit=-1;          //next cycle
                } else last_excit=input.time;
        }//if excitatory
}//Excitatory_CD
```

  - Inhibitory case

    Output spike is generated whenever two spikes occur within the coincidence window and the spike from inhibitory fiber comes first.

```
last_inhib=-1;          //time of the last unused inhibitory spike
                        //-1 when no previous spike is available

Inhibitory_CD(spike){                           //coincidence detection function
                                                //input are spikes

        if (spike.type == inhibitory) {last_inhib=spike.time; return;}
        if (last_inhib == -1) return;           //wait for inhibitory spike

        if (spike.type == excitatory){          //detection
                if (spike.time-last_inhib < w_CD)
                        CD_Fire_Output_Spike(spike.time);
                last_inhib=-1;                  //next cycle
        }//if
}//Inhibitory_CD
```

- Ideal observer

  - The regular spike generation with basic set of parameters

    Fitted interpolation curve $F(D_{ITD}) = 56\sin(3800(D_{ITD} + 0.00009)) + 50.2$ Hz

  - For Meddis model (fiber with CF=140 Hz):

    140 Hz pure tone stimuli $F(D_{ITD}) = 2\sin(3100(D_{ITD} + 0.000226)) + 18.8$ Hz

    White noise stimuli $F(D_{ITD}) = 27\sin(3300(D_{ITD} + 0.000116)) + 19.8$ Hz